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ABSTRACT

This Teacher Resource Manual (TRM) is designed to help teachers implement the Mathematics 20, 23, and 24 courses. The TRM is a support document that provides helpful information to classroom teachers. An overview of the Course of Studies for Mathematics 20/23 and 24 is provided at the beginning of each course section in the TRM. The overview includes the specific learner expectation, the attitudinal expectations and the problem-solving expectations for each course. In addition, the Mathematics 24 overview includes the specific numeration expectations. For each concept covered in every program the skills, process/problem-solving context with sample problems, elective suggestions, and resource correlation are included. Templates for a binomial grid and algebra tiles, support resources, samples of student evaluation ideas, ideas for combining writing and mathematics, guidelines for effective media integration, addresses for the Alberta Consumer and Corporate Affairs Offices, and a list of the Alberta mathematics consultants are appended. (KR)

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Curriculum Branch

SENIOR HIGH
MATHEMATICS 20/23/24

Interim Teacher Resource Manual

1990

Alberta
EDUCATION

SENIOR HIGH

MATHEMATICS 20/23/24

INTERIM TEACHER RESOURCE MANUAL

Curriculum Branch
1990

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PURPOSE

This Teacher Resource Manual (TRM) is designed to help teachers implement the Mathematics 20, 23 and 24 courses. The TRM is a support document that provides helpful information to classroom teachers. The teaching strategies presented are suggestions only and are not legally binding. The Course of Studies for Mathematics 20, 23 and 24 outlines the courses which teachers are legally bound to teach. An overview of the Course of Studies for each of Mathematics 20/23 and 24 is provided at the beginning of each course section in the TRM. The overview includes the specific learner expectations, the attitudinal expectations and the problem-solving expectations for each course. In addition, the Mathematics 24 overview includes the specific numeration expectations.

The remaining portions of the Course of Studies: Program Rationale and Philosophy, General Learner Expectations, Specific Learner Expectations are stated at the beginning of the document and shaded.

The Mathematics 20/23/24 TRM is intended to assist teachers in translating the intentions of the Senior High Mathematics Program into classroom practice.

PROGRAM RATIONALE AND PHILOSOPHY

To set goals and make informed choices, students need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. Emphasis has shifted from the memorization of mathematical formulae and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore. There continues to be a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. Moreover, the use of available technology along with techniques such as estimation and simulation, incorporated with more traditional problem-solving techniques, are the tools by which mathematical problems are solved.

Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require a familiarity with their applications. Most important, students have to be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter high school exhibit mainly concrete operational behaviours with regard to mathematics. It is recognized that high school mathematics courses include many abstract understandings which students are expected to acquire. The course content of the high school mathematics program is cognitively appropriate for the students and should be presented in a way which is consistent with the students' ability to understand.

The Senior High School Mathematics Program includes three course sequences which are Mathematics 14/24, 13/23/33 and 10/20/30. Transfer by students among courses of different sequences is possible. The course sequences commensurate with differing abilities, interests and aspirations, are designed to enable students to have success in mathematics. As well, the mathematics program reflects the changing needs of society, and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

GENERAL LEARNER EXPECTATIONS

Upon conclusion of their senior high school mathematics education, students will be expected to be mathematically literate. **Mathematical literacy** refers to students' ability and inclination to manage the demands of their world through the use of mathematical concepts and procedures to communicate, reason, and solve problems. More specifically, *students will be expected to:*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand the connections and interplay among various mathematical concepts and between mathematics and other disciplines
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

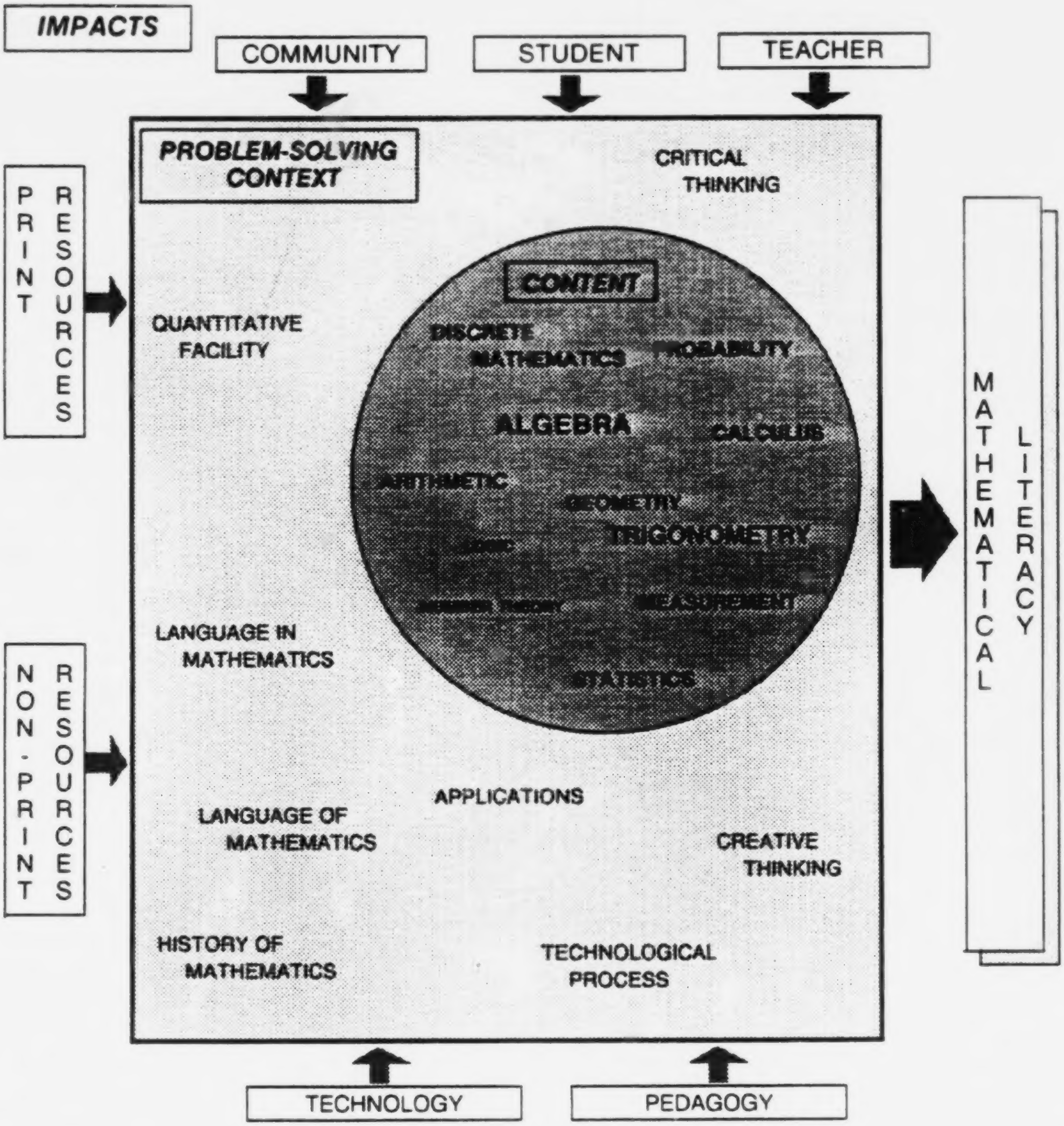
A **General Model for Mathematical Literacy for Senior High School Programs** is presented on the following page which outlines the factors that affect what and how students learn as they become mathematically literate. The model is fluid in that the content can be learned within a problem-solving context which engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces that have impact upon them.

The **Content** of the Senior High School Mathematics Program is the body of knowledge that is to be acquired by students. In the various senior high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

Within each course in the Senior High School Mathematics Program, students will focus on problem solving. The **Problem-Solving Context** refers to the instructional emphases within which the specific content expectations can be acquired. The various entries indicated within the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The **Impacts** on the problem-solving context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent upon the skilful management by the teacher of those items that impact upon a student's learning.

A General Model for Mathematical Literacy for Senior High School Programs


IMPACTS
COMMUNITY
STUDENT
TEACHER
PRINT RESOURCES
NON-PRINT RESOURCES
PROBLEM-SOLVING CONTEXT
CRITICAL THINKING
QUANTITATIVE FACILITY
CONTENT
DISCRETE MATHEMATICS
PROBABILITY
ALGEBRA
CALCULUS
ARITHMETIC
GEOMETRY
TRIGONOMETRY
LOGIC
NUMBER THEORY
MEASUREMENT
STATISTICS
LANGUAGE IN MATHEMATICS
APPLICATIONS
LANGUAGE OF MATHEMATICS
CREATIVE THINKING
HISTORY OF MATHEMATICS
TECHNOLOGICAL PROCESS
MATHEMATICAL LITERACY
TECHNOLOGY
PEDAGOGY

SPECIFIC LEARNER EXPECTATIONS

PROGRAM ORGANIZATION

The major part of the content of each senior high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Each course includes a compulsory component comprising 20% of the course, made up of elective material which is consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Evaluation of students in the Senior High School Mathematics Program will involve assessment of the level of achievement of all of the learner expectations, including concepts, skills and attitudes, as well as problem solving and technology expectations. For more information regarding evaluation, consult the *Teacher Resource Manual for Senior High Mathematics*.

PROGRAM STRUCTURE

At the beginning of each course is a list of attitude expectations. These attitudes embody a mathematical attitude or frame of mind for a student to view the world. The attitude expectations should be woven into the fabric of the entire course.

Following this are the problem-solving expectations that outline a variety of procedures, strategies, skills and checking techniques for solving problems. Because a major purpose for studying mathematics is to learn to solve problems, problem-solving expectations occur throughout all areas of the specific learner expectations. Students must have the background skills and knowledge necessary to achieve these expectations successfully, using problem-solving techniques.

The units of the course are broken into a number of concepts. A concept is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. The concepts are presented as mathematical definitions or theorems or as statements of mathematical ideas or abstractions. In the course of studies, concepts are stated as "*Students will be expected* to demonstrate an understanding that . . ."

Supporting each concept are a number of skills. Skills are intellectual or physical capabilities that will be developed within the context of the particular concept. In the course of studies, skills are stated as "*Students will be expected to . . .*"

Skills specifically related to the use of technology identify areas in which calculators and/or computer technology are applied by students as tools to be used for calculations, manipulation or graphing or to aid in the analysis of problems. Technology expectations are defined explicitly throughout the learner expectations. In many cases, a particular technology is indicated for investigation or analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

Students will be expected to learn how and when to use a calculator and have a demonstrated proficiency in estimation and mental arithmetic. To use calculators effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

Throughout the learner expectations, the words verify and prove appear. For the purposes of the Senior High Mathematics Program, they are interpreted as:

- Verify: to substantiate the validity of an operation, solution, formula or theorem through the use of examples which may or may not be generalized;
- Prove: to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

PROBLEM SOLVING

Through the use of problems and applications, students will be able to see the need for certain concepts and mathematical procedures, to understand concepts and acquire facility with procedures, and to apply and review concepts and procedures already learned. This emphasis on problem solving in high school mathematics has shifted from finding answers to routine "word problems," to the application of mathematical concepts, skills and processes to develop and discover mathematics within a wide variety of problem situations.

Problem solving has been integrated into all of the courses in the Senior High Mathematics Program and is explicitly mentioned throughout the learner expectations. In addition, specific expectations for problem solving are included at the beginning of each course. It is not intended that problem solving be presented as units of study; however, it is expected that upon completion of the course, students will have acquired all of the listed skills and concepts.

Problem solving is the focus of the concept development as well as the exercise portion, of a lesson. New material should be presented as a problem to be solved using previously acquired skills, concepts and processes. Teachers must be open to accepting non-rigorous strategies such as modeling, working backward and simulations and should emphasize the processes used and the type of thinking in which the student is engaged at least as much as the answer to the problem. Teachers must remember that what may be a problem to some students may be only a routine exercise to others, and should be prepared to adapt their presentations and expectations to reflect this. Above all, teachers must model problem-solving behaviour; students will think problem solving is important if they are encouraged to solve problems by their teachers and if they see their teachers solving problems.

At all levels of problem solving, students should be taught how and provided with the opportunities to ask questions and to analyse the thinking processes with which they tackle a problem. An exercise that provides students with this opportunity is outlined below.

"Archie's Envelopes"
Students work in three situations:

1. A group of four students is involved in solving a problem. Each student is given only one piece of information from the problem, in an envelope. Each student must explain their piece of information to the group. The group must:
   a) identify "the problem";
   b) plan a solution;

c) try out the plan; and,
d) analyse the "plan" and method of solution.
2. Same as situation 1, with two students.
3. Students work individually on solving a problem.
(Thanks to A. Coderre, Math 10 Field Test Teacher, for this idea.)

As students become more adept at problem solving, they will begin to use more complex problem-solving processes. Such higher level processes will include making and testing conjectures, determining valid inferences, using inductive and deductive processes and developing and using algorithms to apply to a particular situation.

Through the course of studies, verbs such as "verify" and "prove" are used. These expectations suggest explicitly to the teacher that a student should have the background skills and knowledge necessary to achieve the expectation successfully using problem-solving techniques. Teachers are encouraged to adhere to the spirit of these expectations by treating them as problems to be solved. Students are more likely to internalize the skills or concepts if they are encouraged to develop them themselves.

TECHNOLOGY

Students will be expected to learn how and when to use a calculator and have a demonstrated proficiency in estimation and mental arithmetic.

To use calculators effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

Throughout the learner expectations, specific applications of calculators and computers are identified. These define explicitly the areas in which students are expected to use technology in their study of mathematics. Many of these expectations indicate that a particular technology be used in an investigation or for analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

The integration of calculators into all courses in the high school mathematics program, in support of instruction, in class work, homework and evaluation, is essential. By using calculators appropriately, time previously spent on computation can be used for problem solving.

The use of computers by teachers and students in mathematics classes should be expanded. Teaching using computers allows concepts to be presented visually through graphics that can be manipulated by the teacher according to input supplied by the students. Students should be taught to use prepared software and to use simple programs written for particular purposes in order to use the power of the computer to create geometric displays, organize, analyse and present data, simulate real-life situations and create and run algorithms, as well as to do symbolic manipulations and calculations.

Electronic technology will continue to play an important role in the teaching of mathematics in both what is taught and how it is taught. Teachers should encourage students to use the available technologies as much and as well as possible and should adapt their teaching techniques and evaluation emphases to reflect this.

PLANNING FOR INSTRUCTION

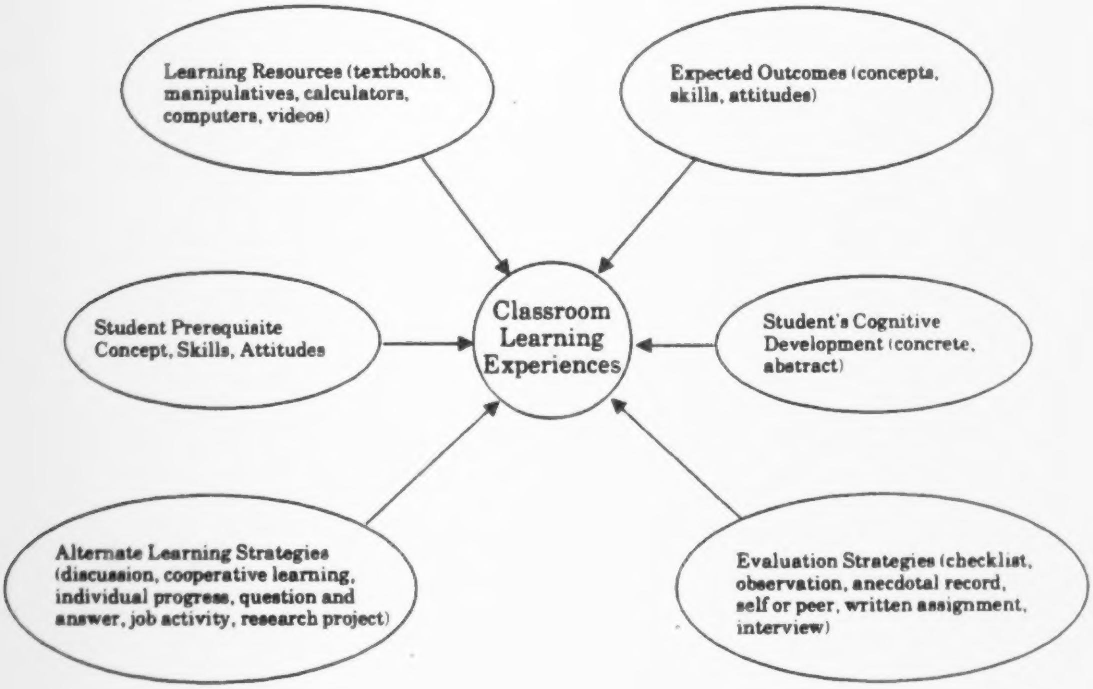
When planning, teachers should begin at the end by identifying what students should know, be able to do, and feel after instruction has occurred. Planning involves determining the destination that teachers want their students to reach and plotting the manner in which the teacher will help students reach that destination. The basic elements of planning are to:

- identify where the students are going (general and specific learner expectations – concepts, skills and attitudes)
- identify how the students will get there (learning strategies)
- identify how you and the students will know when the students have arrived (evaluation strategies).

Planning is a systematic approach to integrating learning experiences to achieve the concept, skill and attitude expectations. However, the instructional plan is a proposal, not a recipe. It should have sufficient flexibility to allow the teacher to relate to students' abilities and to take advantage of learning opportunities that may arise during instruction. The teacher should keep in mind that there are many ways to reach the same destination and not all students require the same learning strategy.

Instructional planning includes yearly, unit and daily lesson planning. The elements used in planning at the three levels are similar but vary in the degree of detail. The elements which should be considered in determining the instructional experiences that the student will receive include: the learner expectations, where the student is in term of these expectations, the cognitive ability of the student, the basic and support learning resources that will be used, alternative learning strategies and how the experience will be evaluated.

Elements of Planning


Learning Resources (textbooks, manipulatives, calculators, computers, videos)
Expected Outcomes (concepts, skills, attitudes)
Student Prerequisite Concept, Skills, Attitudes
Classroom Learning Experiences
Student's Cognitive Development (concrete, abstract)
Alternate Learning Strategies (discussion, cooperative learning, individual progress, question and answer, job activity, research project)
Evaluation Strategies (checklist, observation, anecdotal record, self or peer, written assignment, interview)

When the teacher is determining the sequence in which units and topics are to be presented they should consider the way their students learn and the way the resources they have selected are organized. The way that the concepts, skills and attitudes are organized in the Course of Studies is not necessarily the way that learning will occur.

Long-range course and unit plans should be communicated to students, parents and administrators. Outlines distributed to students should include evaluation criteria, to allow students to assess their own progress and avoid any surprises at report card time.

Learning is enhanced when the expected outcomes of the course match the learning experiences and the methods of evaluation. When a student are clear about where they are going and can see they are making progress toward reaching this end point, they become motivated to continue to work toward attainment of the desired outcomes.

STUDENT EVALUATION

CHARACTERISTICS OF EFFECTIVE EVALUATION OF STUDENT LEARNING IN MATHEMATICS

1. There should be a strong and obvious connection among the learner expectations, the learning experiences in the classroom and methods of evaluation.

   - The activity, test, or assignment must evaluate the student's success in meeting the learner expectations. For example, if the activity evaluates only content recall when the emphasis in the course is on the development of the students' abilities to analyze, interpret and apply concepts, little information will have been produced regarding student mastery of what the student is expected to learn. Evaluation situations which require students to respond to isolated questions on specific learner expectations do not reflect a mathematics curriculum which is an integration of mathematical concepts and procedures in a coherent, meaningful structure.

   - When assessment instruments are aligned with the designed curriculum and the instructional practices in the classroom, a very powerful and consistent message is delivered to the student. What is evaluated and how it is evaluated very clearly tells the student what is important in the course. For example, if the learner expectations stress acquisition of higher order thinking skills, but the evaluation emphasizes only factual recall and the teacher lectures most of the time, students will be somewhat confused over what exactly it is they are expected to be learning but will likely conclude that learning facts is most important. Since students are expected to be able to use calculators, computers and manipulatives, evaluation situations must not only reflect the use of these items but the test items must be appropriate for use with these materials.

   - Learning mathematics extends beyond just learning concepts, procedures and their application. It also includes developing a disposition toward mathematics and seeing mathematics as a powerful way of looking at situations. Dispositions refer not simply to attitudes but extends to a tendency to think and act in positive ways. Students mathematical dispositions are apparent in the way they approach tasks – whether with confidence, willing to explore alternatives, perseverance and interest – and their tendency to reflect on their own thinking. Information about student dispositions to mathematics is best collected through observation of students as they attempt to solve problems.

   - The amount of alignment depends on the extent that the curricular emphasis is reflected in the assessment's relative emphasis. For example, an assessment instrument that contains a large number of questions which ask the student to factor a trinomial and contains few problem-solving questions is poorly aligned with a curriculum that stresses problem solving and reasoning.

2. A variety of evaluation approaches should be used to collect information about students' learning.

   - Learning is a complex process, occurring in many ways and at many levels. Assessment situations should demand different kinds of mathematical thinking as well as present the same mathematical concept or procedure in different contexts, formats and problem situations. Since the mathematics curriculum supports an inquiry approach to instruction, stressing problem solving as the medium through which mathematics is learned and applied, assessment should cover many aspects of a single concept, use procedures in many contexts and require students to show they have integrated their knowledge.

   - No single evaluation instrument or test can provide all the information teachers require to assess what and how their students are learning. The quality of judgments about student attainment of the learner expectations is related to the consistency of the results obtained from a number of different sources. When a student performs similarly on many different tasks, teachers can have confidence in their judgment of that student.

3. Student evaluation requires careful planning and the development of appropriate evaluation criteria.

   - Evaluation is an integral part of planning; it is not something to be considered after instruction. Evaluation should be addressed during all phases of planning and instruction. Evaluation is an integral part of the teaching process, not separate or adjunct to it.

   - Before instruction can begin on a unit of study (year's work, unit or daily lesson) the teacher must determine the degree to which students have mastered the prerequisite concepts, skills and attitudes.

   - During planning the teacher must determine what will be accepted as evidence that the expectations for the unit of study have been acquired by the students. Part of this planning is to determine what standard of achievement the teacher is willing to accept (see section on standards).

   - Upon reviewing the result of an evaluation the teacher may decide to alter instructional strategies.

   - A long-range plan for student evaluation should be developed and communicated to students, parents and administrators early in the school term. This plan should indicate how the students' report card and final grade will be determined including a description of the various components that will be included and the relative weightings of these various components.

4. The purpose of an assessment should determine the kind of evaluation situation which will be used.

   - The assessment of student performance serves many purposes – to identify areas of difficulty for individual students, to gather data for instructional planning, to assign grades or evaluate a program. The purpose should dictate the kinds of questions asked, the methods employed and the uses made of the resulting information.

   - The student, the teacher, the school administrator and the general public all want different kinds of information because they are seeking answers to different questions. For example, the results from standardized tests of general mathematical achievement are not necessarily an appropriate measure of the effectiveness of a specific curriculum. Standardized achievement tests are designed to measure individual student's relative position in a population and must maximize individual differences among students while measuring the common elements of their instruction. As such these tests reflect a student's general achievement, background and prior knowledge and therefore may not measure the amount of learning which has occurred as the result of anyone instructional intervention.

5. Evaluation should be continuous.

   - During instruction teachers are continually making judgments about student progress. As teachers are working with students they are making instructional decisions based on the feedback students are giving them. As teachers are talking they are scanning the students for signs of recognition, they ask questions to determine if students understand the material, they listen to students talk to see if students can verbalize what is being studied, they watch students work together on a problem. These informal observations often provide insights into the thoughts of students and can be a very rich source of information for important instructional decisions.

   - The approaches used to evaluate students' performance must take into account the growth and development of individual students so that the teacher can determine the suitability of the program and make adjustments to it when necessary.

   - The rich data that results from using a variety of assessment methods over time should allow the teacher to make judgments about the progress of the student and communicate this information to others.

6. Evaluation approaches should be manageable for the teacher and provide information for the students and the parents as well as the teacher.

   - Evaluation should be seen as a part of normal teaching activity and not as an additional burden.

   - Each class period is an opportunity for some form of formal evaluation that results in a recorded assessment. However, this does not mean that each student needs to be evaluated on every activity, every period. A workable rotation scheme should be used.

   - It is desirable to use evaluation techniques (such as observation checklists or interviews) that are quickly and easily applied while the students are involved in activities.

   - Accurate and detailed records, including anecdotal comments, should be kept in such a way they are meaningful to students and parents.

   - Students should be encouraged to think about their own progress. Evaluation activities should provide opportunities for students to learn more about mathematics and about themselves.

STANDARDS

Standards in education are statements that clearly and concisely communicate the level of achievement required to attain a specific learner expectation. Standards determine the extent to which learner expectations must be achieved in order to be considered successful or the extent to which students must know the content and have the skills required to pass the course. Learner expectations outline what students are expected to learn. Standard statements establish how well it should be learned.

Often teachers do not consciously establish standard statements but each teacher must determine how difficult to make a test or assignment, how much homework they expect a student to undertake, and what is considered satisfactory student work both within the class and on assignments. Teachers rely on their teaching experience, their training, the experience of other teachers, the level of expectations expressed in basic learning resources, and the reactions of students to set their standards.

In fairness to students each student across the province who receives 50% in a Grade 11 mathematics course should have to meet the same standard of achievement as any other student with 50% in the course. Currently, consistency between classrooms and schools across the province occurs as a result of the commonality of teacher experience. It is Alberta Education's intention to assist teachers with the establishment of standards in mathematics. Last fall Student Evaluation Branch sent out sample standard statements for Mathematics 30 in the Mathematics and Sciences Bulletin 1989–90 School Year. It is expected that the Courses of Study for Mathematics 30 and 33 will contain standard statements.

EVALUATION STRATEGIES

Interview A planned interview with a student or a group of students is an effective technique for assessing knowledge, understanding, thinking style or attitude; for assessing communication skills (e.g., verbal vs. non-verbal ability); and for learning about personal interests. A well-conducted interview will also give students an opportunity to reflect on their own learning.

Interviews should have a definite purpose and both the teacher and the students should be aware of that purpose. The interviews must be planned in advance. Teachers should be prepared to ask additional leading or key questions to guide discussion, to probe for understanding and/or to correct misconceptions.

A one-on-one interview, conducted with a single student, may be used to assess individual students while small group interviews may be used for assessing instructional effectiveness and for consolidating concepts and skills.

Observation Teachers observe all the time. When observations are documented their effectiveness as an evaluation strategy increases immensely. Documented observations often provide the raw data required for analysis and diagnosis, and provide the basis on which to make remediation or enrichment decisions.

Unlike the interview, observation is a passive strategy. The teacher observes students at work, looking for specific behaviours or outcomes. Documentation may occur in the form of anecdotal records or checklists. Some elements that may be monitored are understanding of concepts or generalizations (e.g., when using manipulatives or in group discussions), enthusiasm, willingness to participate or share ideas, perseverance and independence.

Checklist A checklist is a documentation strategy and is used in conjunction with other evaluation strategies. Checklists can easily be created and customized to meet many different needs and situations. Generally, a matrix is created, listing indicators of desirable behaviours or outcomes on one side, and listing ratings, skill levels or evaluative comments along another side. As teachers note a particular behaviour, they need only check the appropriate column that evaluates or rates that behaviour.

Checklists lend themselves very well to documenting such elements of the program as understanding of concepts using manipulatives; mastery of knowledge, skills or objectives; work habits; organizational skills; problem-solving strategies; and cooperative skills.

Anecdotal Records

Anecdotal records refer to the spontaneous documentation of notable behaviour, effort, achievement, attitudes, changes in performance, social skills or communication skills. Records may be kept in such locations as a daily or weekly diary, individual student files, a specific location in the marks record book, or in a common file that serves as a collection of short dated notes.

Anecdotal records provide specific and dated information that can form the basis for conclusions and assessments. These records often prove invaluable in clarifying assessments and add credibility to observations and recommendations being offered in student, parent and/or teacher meetings.

Written Assessments

Traditional multiple choice and open-ended paper-and-pencil assessments fall into this category.

Two other types of written assessments should be used. One is project writing where students may report on a particular interest or research. The evaluative technique and grading system used in this case is not unlike the one used in language arts or social studies.

The second is diagnostic writing. Writing assignments that respond to specific mathematics questions, in an expressive writing style, have proven successful as a diagnostic tool in mathematics. Written responses often force students to examine their own understanding of concepts and will communicate to teachers how much students really know about a concept. Written responses also provide insight to how much understanding students have and how they think. For example, students may be asked to give a non-mathematical example of the inverse rule ($a - {+b} = a + {-b}$). The response will give teachers different information about student understanding than will ten mathematics questions like $3 - 4 =$ __. Written responses can be kept in a student diary or logbook, and may be assigned on a regular basis (once or twice a week), or in lieu of a regular quiz.

Peer Evaluation and Self-Evaluation

Involving students in evaluation involves them in their learning. In self-evaluation, the student reflects on their own learning and can come to establish goals for their own improvement. Skills in self-evaluation are important for students for when they leave a classroom environment. Conferences can include self-evaluation. Teacher questions can be effective for initiating self-evaluation (How did you go about solving this problem? What are the strengths of your solutions?).

Peer evaluation gets students involved in finding value in the mathematical task and developing a sense of problem solving as collaborative work. Checklists are useful to focus peer evaluation and self-evaluation.

SAMPLES OF STUDENT EVALUATION IDEAS

For samples of student evaluation ideas, see Appendix C.

WRITING IN MATHEMATICS

Communication, both written and oral, plays a vital role in the implementation of this mathematics program. Students should not only be involved in group problem-solving activities where they must share ideas with one another but should also have the opportunity to express their ideas in writing.

Writing in mathematics requires students to reflect on their reasoning, enhancing cognitive development. Writing also helps the teacher: more than just filling in answers, it reveals the student's level of understanding. Since you will have a good idea of where your students "are" (in their understanding), you will have the background information necessary to plan lessons that promote mathematical thinking and understanding.

This type of writing - writing to describe thinking and reasoning - will be new to students. Hence, students will need to spend time in going "beyond the right answer." This emphasis on writing will help our students obtain the goal of mathematical literacy as it promotes thinking and understanding about how a problem is solved.

For more information and examples of writing in high school mathematics, see Appendix D.

COURSE SEQUENCES

To address the diverse needs, abilities and interests of students, the Senior High Mathematics Program is composed of four sequences. Each of the sequences has the development of mathematical literacy as a primary goal and presents curricular content within a problem-solving context.

MATHEMATICS 16/26

The Mathematics 16/26 sequence is designed for students in the Integrated Occupational Program. Upon successful completion of Mathematics 16, students will have fulfilled the mathematics requirement for qualification for the Certificate of Achievement.

MATHEMATICS 14/24

The Mathematics 14/24 sequence is designed for students whose needs, interests and abilities focus on basic mathematical understanding. The emphasis is on the acquisition of practical life skills and students are provided with opportunities to improve their skills in working with mathematics. Students who successfully complete Mathematics 24 may choose to enter directly into a job or select from a limited number of trade programs. The mathematics requirement for the General High School Diploma consists of two courses in mathematics; e.g., Mathematics 14 and Mathematics 24.

MATHEMATICS 13/23/33

The Mathematics 13/23/33 sequence covers material from the areas of algebra, geometry, trigonometry, statistics and consumer mathematics.It is designed for students who require mathematics to prepare them for community and technical colleges, trades and employment. Upon successful completion of Mathematics 23, students will have fulfilled the mathematics requirement for qualification for the General High School Diploma.

MATHEMATICS 10/20/30

The Mathematics 10/20/30 sequence is designed for students with an interest and aptitude in mathematics, who are intending to pursue post secondary studies at a university or in a mathematics-intensive program at a technical school or college. Mathematics 10/20/30 emphasize the theoretical development of topics from the areas of algebra, geometry, trigonometry and statistics up to a level acceptable for entry into universities and other post secondary institutions. Upon successful completion of Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

MATHEMATICS 31

This course is designed for highly motivated, academic students who wish to pursue a first course in calculus and vectors. Mathematics 30 must be taken as a pre- or a co-requisite course.

COURSE SEQUENCE TRANSFER

According to their interests, needs and abilities, students who have been promoted to high school may choose to begin their high school mathematics education by enrolling in any of the Grade 10 mathematics courses. Some students may wish to complete a basic Grade 10 mathematics course in order to increase the likelihood of success in a more challenging course. The provision of course transfer enables students to change course sequences, when necessary, to more appropriate levels of challenge.

A student who has achieved a mark of 50% or higher in a given course shall be eligible to take the next or higher ranking course in that sequence.

The local school authority shall have a policy that states clearly the criteria to be met by a student who wishes to change program routes.

When counselling a student to use the course transfer provisions the following circumstances should be considered:

- the student's ability and motivation
- the best interest of the individual student
- judgments are made on an individual basis, not for an entire class of students.

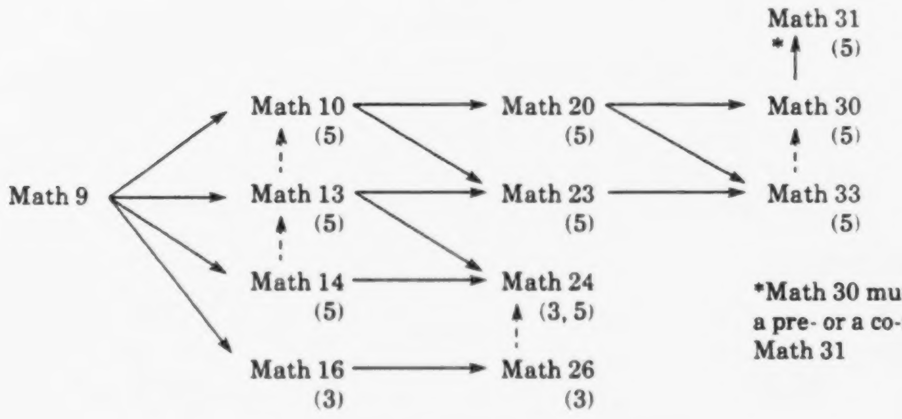
When transferring from a 10/20/30 sequence to a 13/23/33 sequence, or from a 13/23/33 sequence to a 14/24 sequence, the student should transfer into the sequence at the next grade level (for example, from Mathematics 20 to Mathematics 33, or from Mathematics 13 to Mathematics 24). When a student transfers into a less academic sequence, the principal may waive the normal prerequisite for students with less than a 50% standing.

Upon successful completion of Mathematics 33, students may wish to enrol in Mathematics 30 in order to qualify for the Advanced High School Diploma. Students seeking to make this type of transfer should be aware of the increase in academic demand in Mathematics 30 and should be able to demonstrate a level of achievement in Mathematics 33 which would make success in Mathematics 30 probable. Although less common, some students may wish to enrol in Mathematics 20 after completing Mathematics 23 or in Mathematics 13 after Mathematics 14 or in Mathematics 24 after Mathematics 26. The school principal must waive the normal prerequisite for students wishing to make this type of transfer. Students with credits in a course at lower grade level are not eligible for credits for waived prerequisite courses. For example, students who complete Mathematics 13/23/33 and successfully complete Mathematics 30 are not eligible for credits for the waived prerequisite courses Mathematics 10 and 20 unless they actually take the course.

Please consult the *Guide to Education: Senior High School Handbook 1990–91* (p. 52–54) for further information on course sequence transfer and awarding of retroactive credits.

Provincially developed programs have been designed to accommodate transfer between course sequences at particular points. In the following chart, normal prerequisites are indicated by a solid line and recommended transfer points are indicated by a broken line. Special circumstances may warrant student transfer at other points in the curriculum. All decisions about student transfers between course sequences are made at the local school level since this is where the best interest of the individual student can be determined.

Senior High Mathematics – Course Sequence



*Math 30 must be taken as a pre- or a co-requisite to Math 31

(Credit Values Indicated in Brackets)

LEARNING RESOURCES

INTRODUCTION

Learning resources fall into three categories: Basic, Support and Other learning resources. These categories include print, non-print and electronic software materials used by teachers or students to facilitate teaching and learning.

Basic Learning Resources

Basic learning resources are those student learning resources authorized by Alberta Education as the most appropriate for addressing the majority of learning expectations of course(s), substantial components of course(s), or the most appropriate for meeting general learner expectations across two or more grade levels, subject areas, or programs as outlined in provincial Programs of Study. These may include any resource format; e.g., print, computer software, manipulatives or video.

Support Learning Resources

Support learning resources are those student learning resources authorized by Alberta Education to assist in addressing some of the learner expectations of course(s) or components of course(s); or assist in meeting the learner expectations across two or more grade levels, subject areas, or programs as outlined in the provincial Programs of Study.

Other Learning Resources

Other learning resources are those learning resources identified by Alberta Education as useful for teachers in the implementation of a course(s) or Program(s) of Studies, but which have not undergone the standard review procedures of Alberta Education. Alberta Education does not accept responsibility for use of these resources with students. A disclaimer statement is used in teacher support publications whenever these resources are listed.

The following learning resources support the Mathematics 20/23/24 courses. The abbreviations for the resources, used throughout the TRM, are noted in brackets. A description of the Support and Other resources can be found in Appendix B.

MATHEMATICS 20

Basic

Exploring Probability - Student Edition (EP), Newman, C.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

Holtmath 11 (HM11), Bye, M. et al., Toronto, Ontario: Holt, Rinehart and Winston, 1988.

Mathematics 11: Principles and Process (MPP11), Ebos, F. et al., Scarborough, Ontario: Nelson Canada, 1989.

Mathematics 11 National Edition (M11), Kelly, B. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1989.

The Art and Techniques of Simulation - Student Edition (ATS), Gnanadesikan, M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

Ressource de Base

Mathématiques 11, Ebos, F. et al., Montréal, Québec: Éditions Beauchemin ltée, 1988.

Support

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1986.

Algebra Tiles for the Overhead Projector, Howden, H., New Rochelle, New York: Cuisenaire Company of America, 1985.

Algebra Tiles – Student Set.

Curriculum and Evaluation Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

Exploring Probability – Teacher's Edition (EP), Newman, C.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

Holtmath 11 Teacher's Resource Manual (HM11 TRM), Bye, M. et al., Toronto, Ontario: Holt, Rinehart and Winston, 1989.

How to Evaluate Progress in Problem Solving, Charles, R. et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Master Grapher, Demana, F. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1988.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

Of Dice and Men, National Film Board (video).

Quantitative Literacy Software, American Statistical Association.

The Art and Techniques of Simulation – Teacher's Edition (ATS), Gnanadesikan, M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

The Language of Graphs, Swan, M., Nottingham, England: The Shell Centre for Mathematical Education, 1987.

Trigonometric Functions I, TV Ontario, 1987 (video).

Other: see Appendix B.

MATHEMATICS 23

Basic

Applied Mathematics 11 (AM11), Dottori, D. et al., Toronto, Ontario: McGraw-Hill Ryerson Limited, 1988.

Exploring Probability – Teacher's Edition (EP), Newman, C.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

Mathematics for a Modern World Book Three (MMW2), Carli, E.G. et al., Toronto, Ontario: Gage Educational Publishing Company, 1985.

Math Matters Book 3 Alberta Edition (MM3), Ebos, F. et al., Scarborough, Ontario: Nelson Canada, 1991.

Ressource de base

Mathématiques pour un monde moderne, livre 3 (MPMM3), Baxter, W.H. et al., Saint-Laurent, Québec: Éditions du Trecarre, 1989.

Support

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virginia: The National Council of Teachers of Mathematics, Inc., 1986.

Algebra Tiles for the Overhead Projector, Howden, H., New Rochelle, New York: Cuisenaire Company of America, 1985.

Algebra Tiles: Student Set.

Applied Mathematics 11 Teacher's Guide (2nd Edition), McPhail, D., Toronto, Ontario: McGraw-Hill Ryerson Limited, 1990.

Curriculum and Evaluation Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

Exploring Probability – Teacher's Edition (EP), Newman, C.M. et al., Palo Alto, California: Dale Seymour Publications, 1987.

How to Evaluate Progress in Problem Solving, Charles R., et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Master Grapher, Demana, F. et al., Don Mills, Ontario: Addison-Wesley Publishers Ltd., 1988.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

Of Dice and Men, National Film Board (video).

Quantitative Literacy Software, American Statistical Association.

The Language of Graphs, Swan, M., Nottingham, England: The Shell Centre for Mathematical Education, 1987.

Trigonometric Functions I, TV Ontario (video).

Other: see Appendix B.

MATHEMATICS 24

Basic

Consumer and Career Mathematics Second Canadian Edition (CCM), Bolster, L. et al., Toronto, Ontario: Gage Educational Publishing Company, 1991.

Consumer Mathematics (CM), Lange, W.H. et al., Markham, Ontario: Houghton Mifflin Company Limited, 1985.

Support

Activities for Implementing Curricular Themes from the Agenda for Action, Hirsch, C.R. (ed), Reston, Virgina: The National Council of Teachers of Mathematics, Inc., 1986.

Curriculum and Evaluation Standards for School Mathematics, National Council of Teachers of Mathematics, Reston, Virginia: National Council of Teachers of Mathematics, 1989.

How to Evaluate Progress in Problem Solving, Charles, R. et al., Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

Mathematics Dictionary Fourth Edition, James, G. et al., New York, New York: Van Nostrand Reinhold Company, 1976.

Other: see Appendix B.

HOW TO USE THE TEACHER RESOURCE MANUAL

This Interim Teacher Resource Manual is designed to assist teachers in implementing the Mathematics 20/23/24 courses. It is specifically designed to be used on a regular basis. The design of the manual is as follows:

CONCEPT 1

The concept box includes the specific learner expectations taken from the course of studies.

COMMENTS	The **comments** section includes: comments to help clarify the intent of specific learner expectations; an overview of the material that students may have covered in previous courses; notes regarding material that may be considered extraneous; and limiting examples.
PROCESS/PROBLEM-SOLVING CONTEXT	Suggestions for activities and examples of problems that are appropriate to the specific learner expectations are included, as well as references to other books and articles applicable to the specific learner expectations. Periodically, there will be problems or activities that apply to specific learner expectations; they will be preceded by **"Activity/Problem 2.2"** (this activity or problem refers to learner expectation 2.2). If the comment is preceded by **"Activity/Problem"** then the comment refers to the whole concept.
TECHNOLOGY INTEGRATION	Suggestions for specific calculator and computer exercises, video programs, and for authorized computer software are included.
ELECTIVE SUGGESTIONS	In some cases, elective suggestions are provided. This includes problems and activities that are extensions of the specific learner expectations.
RESOURCE CORRELATION	Provides a correlation of the Basic and Support textbooks that are approved for the Mathematics 20/23/24 courses.

MATH 20

PROGRAM EMPHASIS

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of their required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 20

Unit	Suggested Program Emphasis (in %)
Radicals and Exponents	10
Rational Expressions	10
Probability	8
Functions and Relations	16
Quadratic Functions	8
Quadratic Equations	8
Geometry	8
Trigonometry	12
Elective	20

ATTITUDES

Students will be expected to demonstrate an attitude associated with mathematical literacy. In particular, ***students will be expected to***:

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- be critical and constructive in approaching new ideas and new processes;
- be aware of the importance of communication skills in mathematics;
- appreciate the usefulness of computational competence, mathematical processes and problem-solving skills which are used in the decision making and modelling processes in our society;
- appreciate the contributions of mathematics to our culture and civilization.

PROBLEM SOLVING

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to:

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to:

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns
- work backward
- break the problem into smaller parts
- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations

- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to:

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilize appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to:

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

RADICALS AND EXPONENTS
PROGRAM EMPHASIS - 10%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding of the relationship between radical expressions and exponential expressions with rational exponents and that these expressions are subject to the laws of exponents.	Students will be expected to transform expressions from radical to exponential form and vice-versa. Students will be expected to simplify expressions which are written in radical or exponential form.	Students will be expected to evaluate radical or exponential expressions using a calculator.
Students will be expected to demonstrate an understanding that radical expressions and equations which contain radicals can be evaluated and simplified by performing basic arithmetic operations.	Students will be expected to change the form of square root expressions from mixed to entire radicals and vice-versa. Students will be expected to perform the operations of addition, subtraction, multiplication involving square root expressions. Students will be expected to solve radical equations.	Students will be expected to solve problems involving radical equations.

CONCEPT 1

Students will be expected to demonstrate an understanding of the relationship between radical expressions and exponential expressions with rational exponents and an understanding that these expressions are subject to the laws of exponents.

Students will be expected to:

1.1 transform expressions from radical to exponential form and vice-versa
1.2 simplify expressions which are written in radical or exponential form
    1.2.1 evaluate radical or exponential expressions using a calculator

COMMENTS

The intent of this concept is for students to understand the relationship between rational exponents and radicals. In Grade 9 students determine the square root of perfect square numbers, in Grade 10 students determine the square root of non-perfect squares - the expectation here extends the relationship into the cube, fourth, fifth and any root of any number. Your students may require a review of previous work (particularly number systems and exponent laws) prior to introducing the new concept – particularly when discussing the need for rational exponents. This expectation is extending the concepts developed in Mathematics 10 dealing with the Real Number system.

This concept provides an opportunity to demonstrate the proper use of the calculator – students should see that they can use both the y^x key as well as the $\sqrt[x]{\ }$ key when evaluating exponential and radical expressions – the use of the calculator does not limit the value of the base. This is also a good time to reinforce the students' estimation skills – i.e. what do you think the cube root of 8 is? – and then evaluate the cube root of 8 on a calculator. More extreme examples could be: What is the value of $6^{7/3}$? Students could estimate the value and then use the calculator to find the value.

Activity: Have students use their calculators to examine the relationships between the base and the exponent – for example, for what values of the exponent can you find the root of a negative number?

Problem: Express in simplest form:

$3 \bullet (8)^{2/3} \bullet (27/8)^{-1/3}$

Answer: 8

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: A certain sample of radium is decreasing according to the equation $A = 3(2^{-t/1800})$, where t is in years and A is in milligrams.
(a) How much radium was in the sample at $t = 0$.
(b) How much will there be 900 years later?

Answers: (a) 3 mg (b) $\frac{3\sqrt{2}}{2}$mg or ≈ 2.1 mg

Reprinted from delta-K, April 1989, "Mathematics for Gifted Students," p. 44.

Note: Prior to solving the problem, have students explore the meaning of the negative exponent and the "$t/_{1800}$" in the equation. Once the students have obtained the answer, have them discuss the implication of their answer: only approximately 0.9 mg is gone after 900 years. How long do they think it would take to have the sample decrease to half its size? How long would it take to be completely gone?

Problem: If $\sqrt[4]{10} \approx 1.7783$, find the approximate value of the following without the use of a calculator
(a) $10^{5/4}$
(b) $10^{-3/4}$

(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem: Does $\sqrt{2} + \sqrt{3} = \sqrt{5}$? Why or why not?

Note: This is a good introduction to operations on radicals. You may wish to extend this to subtraction and multiplication.

For an example of integrating the spreadsheet into mathematics activities, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988.

Problem: Using your calculator find an approximate value for x in: $3^x + 3^x + 3^x = 25$. Explain the approach that you took to solve this problem.

Answer: Approximately 1.93

(Thanks to M. Ray, J.A. Williams H.S.)

Problem: Which is larger, the tenth root of ten or the cube root of two?

Answer: cube root of two

Reprinted from Mathematics Teacher, National Council of Teachers of Mathematics, December, 1987.

Problem 1.2: Which is larger $\sqrt{10} + \sqrt{17}$ or $\sqrt{53}$?

Solve the problem using the calculator.

Answer: $\sqrt{10} + \sqrt{17}$

Reprinted from Mathematics Teacher, National Council of Teachers of Mathematics, December, 1987.

ELECTIVE SUGGESTIONS

Problem 1.2.1: If the intensity of sunlight is reduced by $\frac{1}{2}$ at a water depth of 1 m, what is the intensity at a depth of 3.5 m?

Answer: $(\frac{1}{2})^{3.5}$

Reprinted from "Application in Elementary Algebra and Geometry." Applications in School Mathematics, copyright 1979 by the National Council of Teachers of Mathematics.

Problem 1.2.1: Find the sum:
$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^{10}}$

Answer: $\frac{1023}{1024}$

Encourage students to look for a pattern with smaller series. For example:

$\frac{1}{2^1} = \frac{1}{2}$
$\frac{1}{2^1} + \frac{1}{2^2} = \frac{3}{4}$
$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} = \frac{7}{8}$

A pattern showing that the sum to n terms is $\frac{2^n - 1}{2^n}$ emerges. Check the results using a calculator.

Reprinted from Mathematics Teacher, National Council of Teachers of Mathematics, September, 1986.

RESOURCE CORRELATION

M11:	pp. 16–19, 51–60
HM11:	pp. 100–101, 110–111
MPP11:	pp. 105–109
Mathématiques 11:	pp. 91–96

CONCEPT 2

Students will be expected to demonstrate an understanding that radical expressions and equations which contain radicals can be evaluated and simplified by performing basic arithmetic operations.

Students will be expected to:

2.1 change the form of square root expressions from mixed to entire radicals and vice-versa

2.2 perform the operations of addition, subtraction, and multiplication involving square root expressions

2.3 solve radical equations

 2.3.1 solve problems involving radical equations

COMMENTS

The term "expressions" is used to denote both rationals and variables within the square root - work with the variable should only be included after work is done with integers as it is difficult for students to grasp the concept of $\sqrt{x^2}$.

Historically, the reason for studying radicals was to make the estimation of rational calculations easier. Simplifying radicals from entire to mixed is advantageous in estimating the roots of large radicands - for instance, $\sqrt{24}$ is difficult to estimate (one would have to remember the approximations of a number of the roots for many numbers), but $2\sqrt{6}$ can be estimated easily. This was also the reason for rationalizing the denominators of radicals - for instance $^{2\sqrt{3}}/_{3}$. Again, this is difficult to estimate but $\frac{2\sqrt{3}}{3}$ is easier. It is also important to mention that $\sqrt{3}$ is an exact value and that the decimal value is only an approximation.

Note: Rationalizing denominators is NOT included in this concept.

Specific to expectation **2.3 and 2.3.1**, it is expected that students will solve radical equations that contain only one square root expression.

With the advent of technology the value of performing basic operations on radicals is being questioned by mathematics educators.

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PROCESS/PROBLEM-SOLVING CONTEXT

Problem 2.3.1: For what values does the $\sqrt{2x}$ equal $x\sqrt{2}$?

Answer: $x = 1$ or $x = 0$

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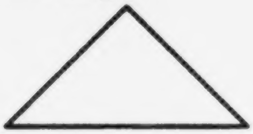
Problem: An equilateral triangle has as many centimetres in its perimeter as square centimetres in its area. What is the length of a side of the triangle?



Answer: $s = 4\sqrt{3}$ cm or ≈ 6.9 cm

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Problem 2.2:

(a) Is $\sqrt{65} \times \sqrt{63}$ less than, equal to, or greater than 64? Estimate the value of $\sqrt{65} \times \sqrt{63}$ in order to make a prediction prior to calculation.

Answer: $\sqrt{65} \times \sqrt{63} < 64$

(b) Is $\sqrt{64} + \sqrt{63}$ less than, equal to, or greater than $2\sqrt{64}$? Estimate the value of $\sqrt{65} + \sqrt{63}$ in order to make a prediction prior to calculation.

Answer: $\sqrt{65} + \sqrt{63} < 2\sqrt{64}$

(c) Given the results of (b), what would you conjecture to be the case for $\sqrt{n+a} + \sqrt{n-a}$, in relation to $2\sqrt{n}$, where $0 < |a| \leq n$?

Provide a logical argument to support your conjecture. Hint: You may wish to try several examples.

Answer: $\sqrt{n+a} + \sqrt{n-a} < 2\sqrt{n}$.

Problem 2.1: Find $\sqrt{16x^{16}}$

Answer: $\pm 4x^8$ Why is the answer $\pm 4x^8$?

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1987.

Activity 2.1: Try this radical approach to handling radicals. Consider, the expression $\sqrt{6\,{}^{6}/_{35}}$.
What would happen to its value if we pulled the whole number out from underneath the radical sign?

For anyone who has had some experience in operating on radical expressions, it doesn't seem possible that $6\sqrt{{}^{6}/_{35}} = \sqrt{6\,{}^{6}/_{35}}$.

Evaluating both expressions with a calculator, however, suggests that they are equal.

Is this true for the square root of every mixed number? Why is it true of the given expressions? Some exploration reveals that the trick works only with expressions of the form $\sqrt{x + {}^{x}/_{(x^2 - 1)}}$.

Discovering and verifying this is a lot of fun.

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Problem 2.2: Explain why $\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$? Are there any values of a and b such that $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$?
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem 2.3: If $\sqrt{x} = y^{2/3}$ and $\sqrt{y} = 2\sqrt{2}$, determine the value of $x + y$.
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem 2.3: Solve $\sqrt{6} = \sqrt{x} + \sqrt{3x+1} + \sqrt{x} - \sqrt{3x+1}$.
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem 2.3: Determine the value of $\sqrt{x} - \sqrt{x - 3}$ if $\sqrt{x} + \sqrt{x + 3} = 6$.
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, p. 62, #2, 5, 9.

TECHNOLOGY INTEGRATION

See MPP11: p. 136, "Computer Insight: Formulas with Radicals"

ELECTIVE SUGGESTIONS

Problem: If $4^x - 4^{x-1} = 24$, then find $(2x)^x$.

Answer: $5^{5/2} = 25\sqrt{5}$

One Solution:

$$\begin{aligned} 4^x - 4^{x-1} &= 24 \\ 4^{x-1}(4-1) &= 24 \\ 4^{x-1} &= 8 \\ 2^{2x-2} &= 2^3 \\ \text{and } x &= 5/2 \\ (2x)^x &= 5^{5/2} \\ &= 25\sqrt{5} \end{aligned}$$

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Problem: The difference between two positive numbers $4\sqrt{3}$. The product of the two numbers is 4. What is the absolute value of the difference of their reciprocals?

Answer: $\sqrt{3}$

Let $x > y > 0$

$$\begin{aligned} x - y &= 4\sqrt{3} \\ xy &= 4 \\ |1/x - 1/y| &= |(y-x)/(xy)| \\ &= 4\sqrt{3}/4 \\ &= \sqrt{3} \end{aligned}$$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1988.

Extension: An extension to this topic could include the study of solving radical equations that included the cube root of an expression.

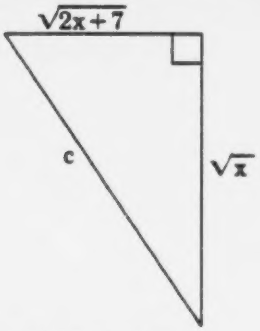
Problem: Solve:

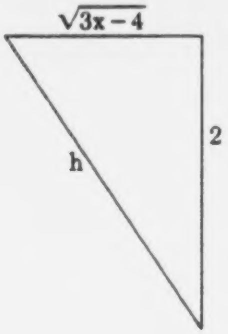
$$\sqrt{16x+1} - 2\sqrt[4]{16x+1} = 3$$

(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem: Find the dimensions of the two triangles, given the following diagrams and that the difference in the lengths of the hypotenuses is 1 cm.

(Thanks to M. Ray, J.A. Williams H.S.)





Possible solution:

$x + 2x + 7 = c^2$

$3x + 7 = c^2$

$c = \sqrt{3x + 7}$

$h^2 = 4 + 3x - 4$

$h = \sqrt{3x}$

$\therefore c > h$

so $c - h = 1$

$\sqrt{3x + 7} - \sqrt{3x} = 1$

$3x + 7 - 2\sqrt{3x}\sqrt{3x + 7} + 3x = 1$

$6x + 7 - 2\sqrt{3x(3x + 7)} = 1$

$-2\sqrt{3x(3x + 7)} = -6x - 6$

$\sqrt{3x(3x + 7)} = 3x + 3$

$\cancel{9x^2} + 21x = \cancel{9x^2} + 18x + 9$

$3x = 9$

$x = 3$

$\therefore c = \sqrt{16}$ and $h = \sqrt{9}$

$= 4$ $= 3$

The dimensions are:

$\sqrt{13}, \sqrt{3}, 4$ and $\sqrt{5}, 2, 3$

RESOURCE CORRELATION

M11:	pp. 20–27, 31–37, 61–63
HM11:	pp. 102–109
MPP11:	pp. 110–127, 130–136

Mathématiques 11: pp. 106–107

RATIONAL EXPRESSIONS
PROGRAM EMPHASIS – 10%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that rational expressions are fractions in which numerators and denominators are polynomials and that operations can be performed on such expressions.	Students will be expected to determine the non-permissible replacement values for the variable in rational expressions in one variable.	Students will be expected to use a calculator or computer to graph rational functions to determine non-permissible values.
	Students will be expected to simplify rational expressions by factoring.	
	Students will be expected to perform the operations of multiplication and division on rational expressions.	
	Students will be expected to perform the operations of addition and subtraction on rational expressions.	
	Students will be expected to solve equations involving rational expressions.	Students will be expected to solve problems which can be represented by equations containing rational expressions.

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CONCEPT 1

Students will be expected to demonstrate an understanding that rational expressions are fractions in which the numerators and denominators are polynomials and that operations can be performed on such expressions.

Students will be expected to:

- 1.1 determine the non-permissible replacement values for the variable in rational expressions in one variable
  - 1.1.1 use a calculator or a computer to graph rational functions to determine non-permissible values
- 1.2 simplify rational expressions by factoring

COMMENTS

Students have not done any previous work with rational expressions. They have solved equations with the variable in the denominator - but this is all. Prior to beginning this unit you may wish to review the methods of factoring. In Math 10 students study the following methods of factoring: common factor; difference of squares; perfect squares; $ax^2 + bx + c$ where $a,b,c \in I$. Note that the sum and difference of cubes is not included and **should not be included in the study of rational expressions.** The sum and differences of cubes will be studied in Mathematics 30 when studying polynomial functions.

The use of a factor of -1 should also be reviewed i.e., that $\frac{(a-b)}{(b-a)} = -1$.

Using the calculator or computer to determine the non-permissible values adds an increased understanding of what it means for the rational expression to be invalid at a certain point or points. Seeing the visual will help students understand why we need to state the non-permissible values - that the graph does not exist at a certain point or points. Students will have discussed prior to this that the denominator of a fraction cannot be equal to zero because it is undefined (the visual adds credence to this).

Note: Students should identify non-permissible values for rational expressions prior to simplifying: For instance, given

$$\frac{x^2 + x - 2}{x^2 + 3x + 2}$$

$$= \frac{(x + 2)(x - 1)}{(x + 2)(x + 1)} \quad x \neq -2, -1$$

$$= \frac{(x - 1)}{(x + 1)} \quad x \neq -2, -1$$

Making note of the non-permissible values prior to simplifying is particularly important in the future when determining domain. The original expression's graph would not exist at

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: For what replacement of x will $\frac{x-5}{x^3-16x}$ not have a reciprocal?

(Thanks to F. Heinen, Lethbridge Collegiate Institute)

TECHNOLOGY INTEGRATION

Software:

- Master Grapher (Addison-Wesley), "Function Grapher" option.
- Computer Graphing Experiments (Addison-Wesley).
- Factoring Algebraic Expressions, (Mindscape) – review.

RESOURCE CORRELATION

M11:	pp. 85–88
HM11:	pp. 52–53
MPP11:	pp. 78–80
Mathématiques 11:	pp. 70–72

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that rational expressions are fractions in which the numerators and denominators are polynomials and that operations can be performed on such expressions.

Students will be expected to:

- 1.3 perform the operations of multiplication and division on rational expressions
- 1.4 perform the operations of addition and subtraction involving two rational expressions
- 1.5 solve equations involving rational expressions
  - 1.5.1 solve problems which can be represented by equations containing rational expressions

COMMENTS

Note the limitation in 1.4 - students are only required to add or subtract with two rational expressions.

In 1.5, you should limit rational expression equations to no more than a degree of two as students have solved quadratic equations that can be factored in Math 10.

In this work, it is important for students to recognize that rational expressions behave the same as fractions and recognize that rational expressions are generalizations of fractions.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem 1.5.1: If the reciprocal of $x+1$ is $x-1$, then $x =$ ____.

Answer: $x = \pm\sqrt{2}$

Problem: The fraction $(5x-11)/(2x^2+x-6)$ was obtained by adding the two fractions $A/(x+2)$ and $B/(2x-3)$. Find the values of A and B.

Answer: A = 3 and B = -1

This problem also provides a review of solving a system of two equations in two unknowns.

Problem 1.5.1: Encourage flexible thinking in problem solving. For example, solve for

$$\frac{x}{x-1} \text{ if } \frac{1}{2-\frac{x}{1-x}}$$

Answer: $x = 0$.
Many students will go through the routine of solving for x. However, by observation it must be the case that $x/_{1-x} = 0$, and therefore $x/_{x-1} = 0$ (and $x = 0$).

Problem reprinted from "Flexibility and Algebraic Problem Solving." Mathematics Teacher, April 1987.

Problem 1.5: If $x + {}^1/_x = 2$, find the value of the following:

(a) $x^2 + {}^1/_{x^2}$
(b) $x^2 - {}^1/_{x^2}$
(c) $x^3 + {}^1/_{x^3}$

Answers: (a) 2, (b) 0, (c) 2

Some solutions for (a).

(i) $x + {}^1/_x = 2$ **may be transformed into a quadratic equation, and x then solved for ($x = 1$).**

(ii) **Try squaring $x + {}^1/_x$, and see what evolves**

$$(x + {}^1/_x)^2 = x^2 + 2 + {}^1/_{x^2}$$
$$= x^2 + {}^1/_{x^2} + 2$$

But also, $(x + {}^1/_x)^2 = 2^2$
$= 4$
Therefore $x^2 + {}^1/_{x^2} = 2$

Reprinted from Mathematics Teacher. National Council of Teachers of Mathematics, December, 1987.

Problem: To solve the equation $\frac{x-5}{x} = \frac{x-5}{x+1}$, Jake reasoned as follows:

"Since the numerators are equal, the denominators are equal. But this is impossible since x cannot equal x + 1. Therefore, there is no solution." Is Jake correct? If not, what is the error in his reasoning? What is the solution?
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem: Find the value of $(x - {}^1/_x)^2$ if $x^2 + {}^1/_{x^2} = 15$.
(Thanks to F. Heinen, Lethbridge Collegiate Institute)

Problem: The area of a rectangle is represented by the expression $A = {}^1/_{(x-2)^2}$. Find the width of the rectangle if the length is represented by the expression $1 = {}^{(3x-1)}/_{(2x^2+3x-14)}$.
(Thanks to M. Ray, J.A. Williams H.S.)

TECHNOLOGY INTEGRATION

Problem: The product of two numbers is four, and their sum is twelve. have students examine ways to find the sum of the reciprocals of the numbers.

Answer: Two possible approaches are:

a) Set up a system of equations, $xy = 4$ and $x + y = 12$, substitute to reduce to one variable, and solve the resulting equation, $x^2 - 12x + 4 = 0$.

   This may be the approach that many students take. However, this equation does not factor easily, and most students will not yet be familiar with the quadratic formula.

b) Encourage students to look for other, less routine (but not necessarily difficult) strategies. For example, return to the original question. To what does $^1/_x + {}^1/_y$ simplify?

   $^1/_x + {}^1/_y = {}^{(x+y)}/_{xy} = {}^{12}/_4 = 3$

ELECTIVE SUGGESTIONS

Problem: Solve for all real values of x:
${}^{(2x-3)}/_x < {}^{(x+5)}/_x$

Answer: $0 < x < 8$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, March 1988.

RESOURCE CORRELATION

M11:	pp. 89–101
HM11:	pp. 54–61
MPP11:	pp. 81–86, 89–93
Mathématiques 11:	pp. 73–81

PROBABILITY
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number form 0 to 1.	Students will be expected to use the language of probability. Students will be expected to determine the sample space for simple events.	
	Students will be expected to determine the probability of simple events through experiments.	Students will be expected to carry out an investigation to determine the experimental probability of an event.
	Students will be expected to determine the theoretical probability of events which have easily countable sample spaces.	Students will be expected to compare theoretical probabilities to those for the same event generated empirically.
Students will be expected to demonstrate that the probability of a compound event can be determined from the probability of the individual events.	Students will be expected to find the probability of two or more events occurring together by the application of the multiplication law for independent events, $P(A \text{ and } B) = P(A) \times P(B)$, and for dependent events, $P(A \text{ and } B) = P(A) \times P(B \mid A)$.	Students will be expected to solve problems which involve finding the probability of the occurrence of two or more events. Students will be expected to determine experimentally the probability of two or more events occurring together and compare it to the theoretical probability.
	Students will be expected to find the probability of the occurrence of one or the other of two events, A and B by the application of the addition law, $P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events and in general, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	Students will be expected to determine experimentally the probability of the occurrence of events A or B and compare it to the theoretical probability. Students will be expected to solve problems which involve finding the probability of either of two events.
Students will be expected to demonstrate an understanding that simulations are experiments which represent the conditions present in real situations through the use of devices and processes with known probabilities.	Students will be expected to design and carry out simulations involving events that have known probabilities.	Students will be expected to use simulation models to solve problems involving events with known probabilities.
	Students will be expected to design and carry out simulations involving events that have unknown probabilities.	Students will be expected to use simulation models to solve problems involving events with unknown probabilities.

CONCEPT 1

Students will be expected to demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number from 0 to 1.

Students will be expected to:

1.1 use the language of probability
1.2 determine the sample space for simple events
1.3 determine the probability of simple events through experiments
    1.3.1 carry out an investigation to determine the experimental probability of an event
1.4 determine the theoretical probability of events which have easily countable sample spaces
    1.4.1 compare theoretical probabilities to those for the same event generated empirically

COMMENTS

Students have done some previous work with probability - in Grades 8 and 9. It is the intent that probability be introduced and then used to explore how it is used in simulating real world situations. Students should experience the situations first before working with the theoretical calculations.

The study of permutations and combinations should not be included at this point - students will be studying this material in Mathematics 30.

PROCESS/PROBLEM-SOLVING CONTEXT

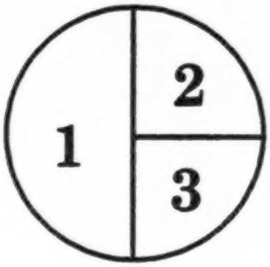
Activity 1: Thumbtacks: Have students speculate on the probability of a thumbtack landing point-up. Give each student a thumbtack and have them flip it ten times, keeping track of "point-up" and "side" occurrences. Record the relative frequency of the "point-up" event for each student, and construct accumulative relative frequency graph. Compare and discuss the speculative and experimental probabilities.

Activity 2: Cylinders: Have students each bring in three sturdy cylinders of various sizes (paper towel tubes, clean metal cans with the labels and ends removed and so on). Mark the ends "heads" and "tails". Have students determine the height to diameter rates and use this value to classify the cylinders. Speculate on the probability of each cylinder landing on its "heads", "tail", or side when flipped. Flip each of the cylinders twenty times, recording the frequencies of the three landing orientation possibilities. Were there any unfair heads/tails cylinders? Is there a pattern of landing emerging related to the ratio classification? What is the height to diameter ratio of a coin? What is the likelihood of it landing on its side?

Adapted with permission from "Experiential Statistics and Probability for Elementary Teachers", in Teaching Statistics and Probability, copyright 1981 by the National Council of Teachers of Mathematics.

Activity: Have the students lay a paper clip on Circle A holding one end of it at the center of the circle with the tip of a pencil. Use the other hand to spin the paper clip 24 times. Record the results in Table 1 below. Divide the totals by 24 to compute the experimental probabilities. Then repeat the entire process using Circle B and Table 2.

A



B

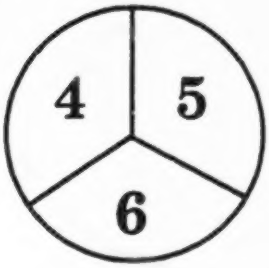


Table 1

Section	Tallies	Totals	Experimental Probabilities
1			
2			
3			
		24	1

Table 2

Section	Tallies	Totals	Experimental Probabilities
4			
5			
6			
		24	1

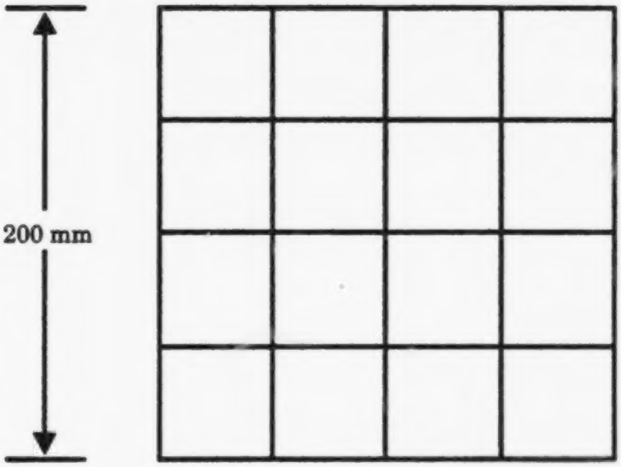
Have the students then determine the theoretical probability of obtaining a 1, 2, 3, 4, 5 or 6 and compare these results with experimental probabilities.

Adapted from "Experimental and Theoretical Probabilities," Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Activity: See "Out of Thin Air," in Activities for Implementing Curricular Themes from the Agenda for Action for a set of activities, using thumbtacks, designed to develop students' understanding of experimental and theoretical probability, and their relationship.

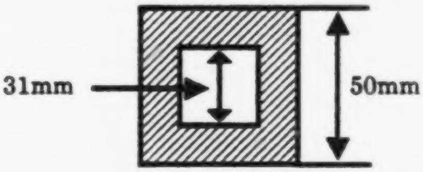
Activity: Create a grid "ruled" in 50mm squares, as shown. Randomly toss 5 pennies on the grid and count the number falling on a line. Repeat the experiments for a total of 20 times. Divide the total number of pennies falling on a line (hits) to the number tossed (100). Use this ratio as an estimation of the probability that a penny, randomly tossed on this grid, will fall on a line.

Total number of hits: ______
Total number tossed: ______
Probability of a hit: ______



Compare the results to the theoretical probability. One approach to determining this value is to consider the centre point of the 19mm penny and lands on a line of the grid. Compare this shaded area to that of the entire square to find the corresponding probability.

P(hit) = $(50^2-31^2)/_{50^2} = 1539/_{2500} = 0.6156$ (approx.)



Adapted from "Probability by Experimentation", Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Problem: Calculate the probability that no two people in the class have the same birthday.

OR

Calculate the probability that two people have the same birthday.

(Thanks to F. Heinen, Lethbridge Collegiate Institute)

TECHNOLOGY INTEGRATION

See MPP11: p. 490, "Computer Insight: The Birthday Problem"

Video:
Of Dice and Men (NFB)

Software:
Quantitative Literacy Software (American Statistical Association): Exploring Probability option.

RESOURCE CORRELATION

M11:	pp. 438–452
EP:	pp. 1–30
HM11:	pp. 350–351
MPP11:	pp. 454–457
Mathématiques 11:	

CONCEPT 2

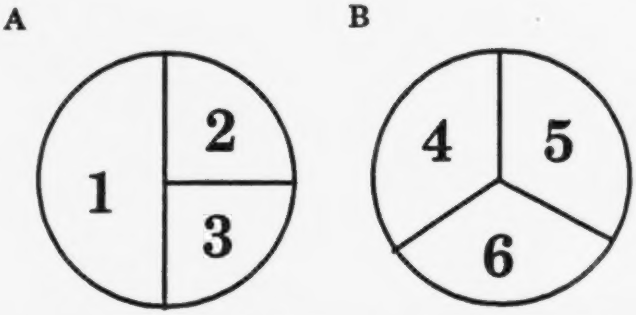
Students will be expected to demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.

Students will be expected to:

- **2.1 find the probability of two or more events occurring together by the application of the multiplication law for independent $P(A \text{ and } B) = P(A) \times P(B)$, and for dependent events, $P(A \text{ and } B) = P(A) \times P(B|A)$**
  - **2.1.1 solve problems which involve finding the probability of the occurrence of two or more events**
  - **2.1.2 determine experimentally the probability of two or more events occurring together and compare it to the theoretical probability**

PROCESS/PROBLEM-SOLVING CONTEXT

Activity:



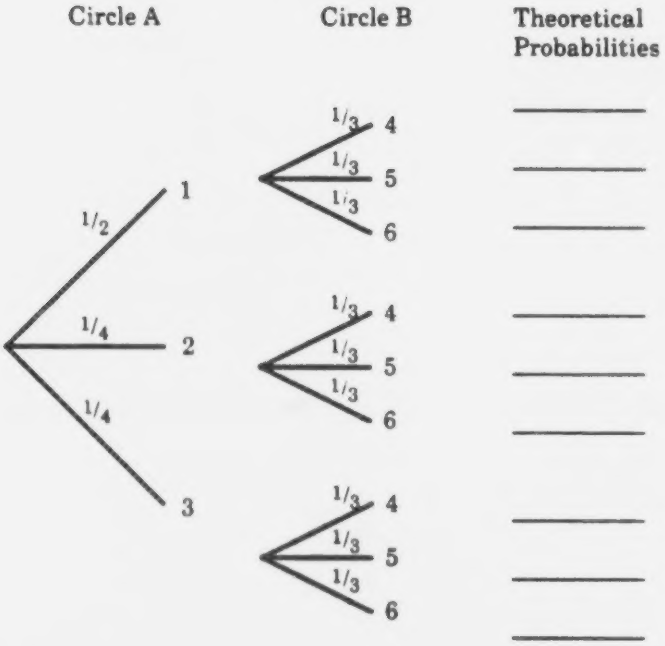
Using a paper clip and a pencil as a spinner as in the activity in Concept 1, page 48, alternate between circles A and B, making 24 pairs of spins. After each pair of spins on Circles A and B, add the two results to get a sum. Record these results in table 3. Again, divide the totals by 24 to compute the experimental probabilities.

Table 3

Section	Tallies	Totals	Experimental Probabilities

Table 4

Sum	Theoretical Probabilities
5	
6	
7	
8	
9	
	1


Circle A
Circle B
Theoretical Probabilities
1/2
1/4
1/4
1
2
3
1/3
1/3
1/3
4
5
6
1/3
1/3
1/3
4
5
6
1/3
1/3
1/3
4
5
6

This tree diagram has nine branches representing the nine different number pairs possible on the two circles, A and B. Multiply the two fractions on each branch to find the theoretical probability for that number pair. Enter these probabilities in the spaces provided. To find the theoretical probability for each possible sum on the two circles, add the appropriate probabilities from above. Enter the results in table 4 above and compare these theoretical probabilities to the experimental probabilities listed in table 3.

Reprinted from "Experimental and Theoretical Probabilities," Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Problem: Each integer 1 through 9 is written on a separate slip of paper, and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer?

A) 0 B) 1 C) 8 D) 9
E) each digit is equally likely

Answer: A) 0

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy Software (American Statistical Association): Exploring Probability option.

ELECTIVE SUGGESTIONS

Problem: A three-person jury has two members each of whom independently has probability p of making the correct decision and a third member who flips a coin for each decision. A one-person jury has probability p of making the correct decision. Which jury has the better probability of making the correct decision if majority rules in the three-person jury?

Answer: Probabilities are the same.
One solution: The two conscientious jurors in the three-person jury agree on the correct decision with a probability of p^2. They will disagree with a probability of $2p(1-p)$, and after consulting the flippant juror, the probability of a correct decision is halved to $p(1-p)$. Hence the overall probability of a correct decision is $p^2 + p(1-p) = p$, the same as that of a one-person jury.

Reprinted from delta-K, April 1989, "Mathematics for Gifted Students," pp. 76 and 102.

RESOURCE CORRELATION

M11: pp. 453–466
EP: pp. 31–42
HM11: pp. 352–354
MPP11: pp. 458–460

Mathématiques 11:

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.

Students will be expected to:

2.2 find the probability of the occurrence of one or the other of two events, A and B by the application of the addition law, $P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events and in general, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

- 2.2.1 determine experimentally the probability of the occurrence of events A or B and compare it to the theoretical probability
- 2.2.2 solve problems which involve finding the probability of either of two events

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy Software (American Statistical Association): Exploring Probability option.

ELECTIVE SUGGESTIONS

Problem: An ordinary deck of playing cards is shuffled and the top card set aside without anyone knowing its face value. What is the probability the next card is a king?

One Solution: Let E be the event of drawing a king on the second card; E_1 the event that the top card is actually a king; and E_2 the event that the top card is not a king.

Then,

$$\begin{aligned} P(E) &= P(E_1)P({}^{E}/_{E_1} + P(E_2)P({}^{E}/_{E_2}) \\ &= {}^{4}/_{52} \times {}^{3}/_{51} + {}^{48}/_{52} \times {}^{4}/_{51} \\ &= {}^{4}/_{52} \end{aligned}$$

See "The Wonder and Creativity" in Mathematics Teacher, September 1988 for further discussion.

RESOURCE CORRELATION

M11:	pp. 427–470
EP11:	pp. 42–50
HM11:	pp. 352–354
MPP11:	pp. 461–463

Mathématiques 11:

CONCEPT 3

Students will be expected to demonstrate an understanding that simulations are experiments which represent the conditions present in real situations through the use of devices and processes with known probabilities.

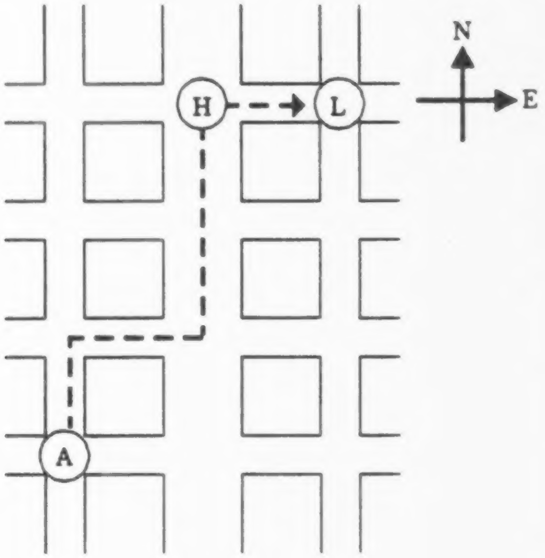
Students will be expected to:

- 3.1 design and carry out simulations involving events that have known probabilities
    - 3.1.1 use simulation models to solve problems involving events with known probabilities
- 3.2 design and carry out simulations involving events that have unknown probabilities
    - 3.2.1 use simulation models to solve problems involving events with unknown probabilities

PROCESS/PROBLEM-SOLVING CONTEXT

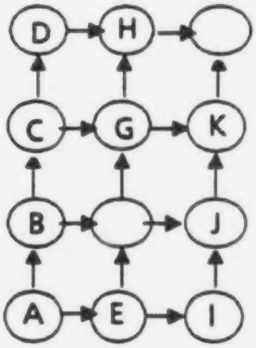
Problem: Probability of a random walk. have the students imagine they are going for a walk in the city, traveling from A to L. At each intersection **where the students have a choice**, they flip a coin to determine whether they go north (heads) or east (tails). What is the probability that they will pass through intersection H?

One possible path is shown, resulting from the four tosses HTHH. There is no choice on the fifth move, so no toss is needed.



Answer: P(H) = $^{5}/_{16}$

Look at a simpler case first. If the students start at A on their way to L, then the probability that they get to B is $^{1}/_{2}$, since they flip the fair coin at A. Like wise, by symmetry, the probability that they get to E is also $^{1}/_{2}$.



The probability (P) that they get to F is the sum of the probabilities of coming from B and from E. For example, P(F from B = $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$. Likewise, P(F from E) is also $\frac{1}{4}$. Thus P(F), the probability that they get to F from B or E, is $\frac{1}{4} + \frac{1}{4}$, or $\frac{1}{2}$.

In a similar fashion, P(C) = P(I) = $\frac{1}{4}$. Since there is no choice of direction in going from I to J, P(J from I) = 1·P(I) = $\frac{1}{4}$

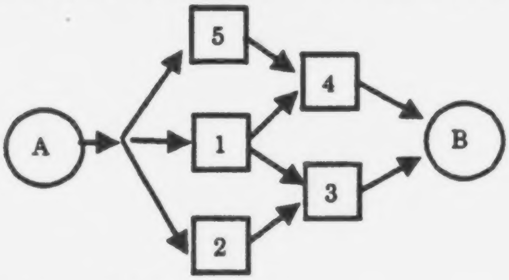
What is P(J from E)? ___

P(D) = ___ P(H) = ___ P(L) = ___
P(C) = $\frac{1}{4}$ P(G) = ___ P(K) = ___
P(B) = $\frac{1}{2}$ P(F) = $\frac{1}{2}$P P(J) = ___
P(A) = 1 P(E) = $\frac{1}{2}$ P(I) = $\frac{1}{4}$

Now complete the missing probabilities for the grid. If the probabilities have been computed correctly, with L the final destination, P(L) should equal 1.

Adapted from "Probability of a Random Walk," Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Activity 3.1.1: Consider the following situation: A section of a city has aging water mains and five pumping stations as shown below. Assume that at a particular fixed time each pumping station has a probability of failure of 1/2. At that time, what is the probability that water will flow from A to B?



The approximate probability is 0.59.

Have students perform a simulation using 0–1 spinners to obtain an estimate of the solution. Give each of five groups a spinner. Each group represents a pumping station; 0 on the spinner represents a pumping failure. After each group spins, determine if a path from A to B exists, recording the result. Repeat this process a number of time, then determine the experimental probability of water flowing from A to B.

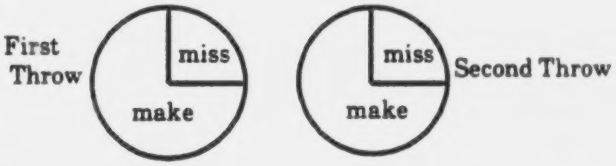
Provide variation by changing the number and/or the configuration of the stations, or the probability of failure (e.g., $^1/_3 - ^2/_3$ spinners).

Adapted with permission from "Random Digits and Simulation," in Teaching Statistics and Probability, copyright 1981, by the National Council of Teachers of Mathematics.

Activity 3.1.1: Consider the following scenario: In the interprovincial girls basketball championship between high schools from Peace River and Regina, the score is tied as time runs out. However, Peace River's best free-throw shooter, with a 0.75 average, is fouled and gets two shots after a short time-out. What, the students wonder, is the probability that she will make at least one shot and win the game?

Possible approaches:

(a) Use two spinners as shown and for thirty pairs of spins, keep track of the number of times at least one spinner lands on "make."



(b) Determine the theoretical probability as
$P = 1 - (0.25)^2 = 0.9375$

Provide variation, for example, by changing the "second throw" spinner to reflect the effects of possible increased pressure and anxiety.

Problem adapted with permission from "Monte Carlo Simulation: Probability the Easy Way," in Teaching Statistics and Probability. Copyright 1981 by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy Software (American Statistical Association): Art and Technique of Simulation Option.

RESOURCE CORRELATION

M11:	pp. 471–479
ATS:	pp. 1–44
HM11:	pp. 355–356
MPP11:	pp.
Mathématiques 11:	

FUNCTIONS AND RELATIONS
PROGRAM EMPHASIS – 16%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that certain observed real-world phenomena are quantitatively related to each other and that theses relations can be described graphically, with sets of ordered pairs, rules and equations.	Students will be expected to graph relations that describe physical phenomena or everyday situations.	Students will be expected to solve problems by graphing and interpreting the graphs that describe physical phenomena and everyday occurrences.
	Students will be expected to determine the domain and the range of relations algebraically and from given graphs.	
Students will be expected to demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).	Students will be expected to represent simple mathematical situations such as direct, inverse and partial variations with tables of values, identify the dependent and independent variables, and express the domain ad range, appropriately noting any restrictions.	
	Students will be expected to interpolate and extrapolate from the graphs of functional relationships.	
	Students will be expected to use functional notation and graphs to describe functional relationships.	Students will be expected to solve problems algebraically or by the use and interpretation of graphs that represent functions.
	Students will be expected to determine those relations that are functions.	Students will be expected to develop and explain tests which could be used to determine if any relation is or is not a function.
	Students will be expected to illustrate and recognize different kinds of functions algebraically and graphically from the following list: linear functions (including identity and constant functions), polynomial functions (including quadratic and cubic functions), reciprocal functions, absolute value functions, and exponential functions.	Students will be expected to draw and analyze the graphs of functions using calculators or computers.

FUNCTIONS AND RELATIONS (CONT'D)
PROGRAM EMPHASIS - 16%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that for some relations called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).	Students will be expected to identify the zeros of a function as the x–intercepts of its graph. Students will be expected to write, and sketch the graphs of, the inverses of relations and functions.	Students will be expected to solve find the zeros of a function by analyzing its graph and its value for various replacements of the independent variable, using calculators or computers.
Students will be expected to demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretching of the graph of any function.	Students will be expected to describe the transformation effects on the graph of $y = f(x)$ of the parameters a and b in $y = f(x-a) + b$. Students will be expected to describe the transformation effect on the graph of $y = f(x)$ of the parameter c in $y = cf(x)$. Students will be expected to describe and sketch the graphs of $y = cf(x-a) + b$ by applying the transformation effects of a, b and c on the graph of $y = f(x)$.	Students will be expected to perform an investigation to determine the effects of the parameters a and b on the graph of $y = f(x-a) + b$. Students will be expected to perform an investigation to determine the effect of the parameter c on the graph of $y = cf(x)$. Students will be expected to predict the graphs of functions written in the form $y = cf(x-a) + b$ given the graph of $y = f(x)$, and verify using calculators or computers.

CONCEPT 1

Students will be expected to demonstrate an understanding that certain observed real-world phenomena are quantitatively related to each other and that these relations can be described graphically, with sets of ordered pairs, rules and equations.

Students will be expected to:

1.1 graph relations that describe physical phenomena or everyday situations
  1.1.1 solve problems by graphing and interpreting the graphs that describe physical phenomena and everyday occurrences
1.2 determine the domain and the range of relations algebraically and from given graphs

COMMENTS

It is the intent of this concept for students to examine relationships and to express these relationships mathematically. The study of functions and relations is fundamental to further studies in mathematics.

In order for students to be adequately prepared for geometry, it is important for students to see and graph the equations of circles – you may wish to extend this to graphing the equations of ellipses.

In the initial exploration of domain and range it will be important for the students to have the graphs of functions and relations to examine. This visual exploration will assist students in determining the domain and range by examining equations (this approach is very abstract).

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students suggest situations where two phenomena are related. For example, the age of a car and its value, the month of the year and the maximum temperature, etc. Discuss how strong the relationships are Can the examples be used with phrases such as "caused by," "depends on," etc.? Have students graph the values to see if they can make any statements about the relationships.

Problems: A glass of water is at room temperature. An ice cube is dropped into it. Draw a graph which describes the temperature of the water over time.

A swimmer is in a race which covers two lengths of a pool. Draw a graph which describes her speed over time.

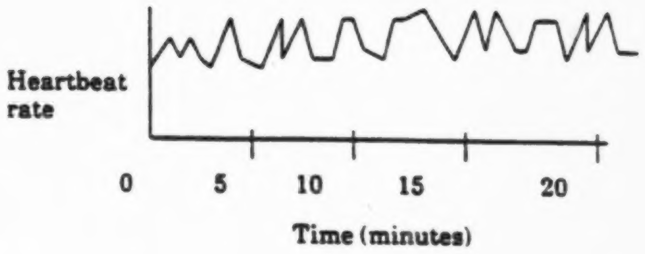
Draw a graph which describes how hungry you are according to the time of day.

Draw a graph which describes the noise level of the seventh game of the Stanley Cup final, from the singing of the national anthem to the end of the game.

You fill an electric kettle with cold water and plug in the kettle. Sketch the graph of the water temperature versus the length of time the kettle is plugged in.

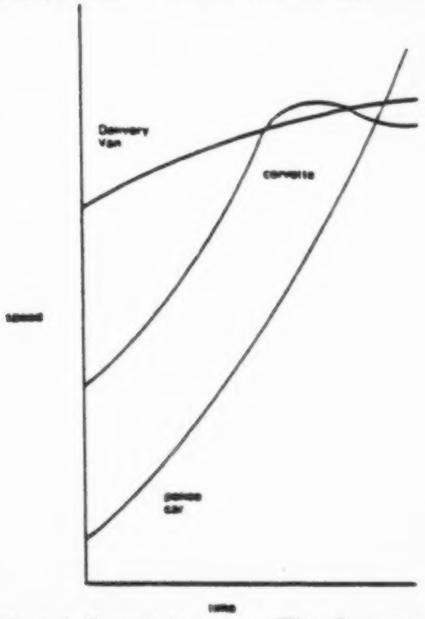
On a sunny day, the length of the shadow of a telephone pole depends on the time of day. Sketch a graph of the length of the shadow versus the time of day.

Problem: The following graph describes the heart rate of a hockey player during the third period of the final game of a championship series.



Can you tell what this player did during the period? When did he play? How much did he play? Was this player in good shape? What could have occurred around the 3-minute mark? the 12-minute mark? Do you think that this player was on the winning team? Why?

Activity: Look at the following graph. It describes the actions of three vehicles, beginning at a busy intersection. Describe what is happening. Who is accelerating/decelerating? When?

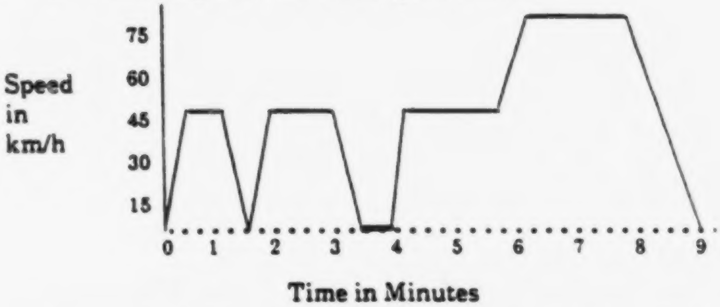


For further information, see The Language of Graphs by Malcolm Swan, pp. 19–47.

Activity: Make up a story to explain the graph.

Connie is driving from home to work. The graph shows her speed at any any time during her nine-minute trip. Make up a story about her trip to explain the graph. In this story tell:

- approximately how far she lives from where she works
- what happened one and one-half minutes after she started
- what happened between three and one-half minutes after she started and four minutes after she started
- something observed about speed limits.



Discuss in detail what the students have written. Are there misinterpretations which need to be clarified?

Reprinted with permission from "Make up a Story to Explain the Graph." Mathematics Teacher, January 1984, copyright by The National Council of Teachers of Mathematics.

For further discussion and activities regarding graph interpretation, see, "Interpreting Graphs," Mathematics Teacher, May 1989.

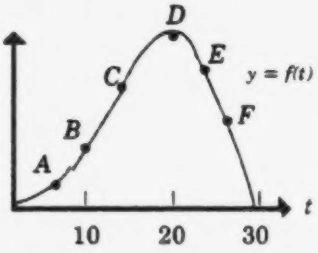
Problem: Peace River Trust, the monthly payments on a loan of $1000 spread over two years are $46.49. The monthly payments on loans of $3000 and $4500 are $139.47 and $209.21 respectively. If Tom wants to borrow $7500 for a used Firebird and Barbara needs $2800 for a dirt bike, what should they expect their payments to be?

The monthly payments on a loan of $1000 change according to the length of time the loan is for. For example, a loan of $5000 over one year has monthly payments of $441.35. Over two years the monthly payments are $232.45 and over five years, the payments are $108.10.

Approximately what would the payments be over four years? over 30 months?

Activity: Have students describe a situation that could be described graphically as a circle.

Problem: A company's profit is a function *f* of the number of years, *t*, that it has existed. At which of the points (A, B, C, D, E, or F) are profits increasing at and increasing rate?



Answer: A

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1987.

Activity: Have students explore the change in area of a square of side length x, and that of one 5 units longer. Consider sketching the situation, writing an expression for this difference of area in terms of x, stating the appropriate domain, and questions such as Will the change of area be less than, equal to, or greater than the area of the original square? (Encourage students to devise ways – e.g., graphing – to answer this question.)

Extend this activity by beginning with a square of constant length of side, for example, 6 units, and describing the change in length of side in terms of x, and (b) by changing one or both diagonals (and thus the shape of the initial square) in terms of x. What function of x now describe the change in area? in domains?

See "Geometrical Adventures in Functionland," Mathematics Teacher, May 1987, for more information and examples.

Activity 1.2: Have students work in groups to sketch the graphs of 10 different relations. Once the graphs have been completed, have students discuss the restrictions of x and y for each relation. Following this, the terms domain and range can be introduced.
(Thanks to D. Rosiewich, M.E. Lazerte H.S.)

TECHNOLOGY INTEGRATION

For an example of how spreadsheets can be used to help develop students' interpretation of relationships between two variables, see "Integrating Spreadsheets into the Mathematics Classroom," Mathematics Teacher, November 1988. Apple Works and Microsoft Works are Alberta Education Basic Resources which provide spreadsheet opportunities.

See "Mathematical Modeling Using Spreadsheets," in delta-K, June 1989, for an example of using a spreadsheet (with graphing capabilities) to better understand functions.

Software:
- Master Grapher (Addison-Wesley)
- Computer Graphing Experiments II (Addison-Wesley)

RESOURCE CORRELATION

M11	pp. 142–159
HM11:	pp. 192–195
MPP11:	pp. 250–253
Mathématiques 11:	pp. 201–209

CONCEPT 2

Students will be expected to demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).

Students will be expected to:

2.1 represent mathematical situations such as direct, inverse and partial variations with tables of values, identify the dependent and independent variables, and express the domain and range, appropriately noting any restrictions

2.2. interpolate and extrapolate from the graphs of functional relationships

COMMENTS

Note the restriction on the study of direct, inverse and partial variations - the reason for this restriction is so that students can graph the relationships. Joint variation is not included as there is at least three variables involved and this would require graphing in three dimension.

"Direct and inverse variations" include the study of direct squared and inverse squared variation.

An alternative approach, using graphs, to solve problems such as "If y varies directly as x and $y = 14$ when $x = 21$, find y when $x = 7$" is to have students examine the graph of $y = {}^{2}/_{3}\,x$ and read from the graph that $y = 14$ when $x = 21$ - then the student would have to read, from the graph the value of y when $x = 7$. To further extend this, students could predict the value of x when $y = 21$ from the graph. This approach reinforces the graphical relationship between x and y and that direct variation is a linear relationship. Once the graphical approach is studied and students see that this relationship occurs, the algebraic approach should be considered as it is difficult to read accurate values from a graph.

For discussion of topics that should be studied in functions, see "Standard 6: Functions," in Curriculum and Evaluation Standards for School Mathematics, National Council of Teachers of Mathematics, 1989.

For a discussion of misconceptions and difficulties students have with the concept of function and suggestions for overcoming or avoiding these problems, see "Difficulties Students Have with the Function Concept," in The Ideas of Algebra, K-12, 1988 NCTM Yearbook.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Present students with the following scenario: Suppose that you have become the manager of a motel, and you wish to investigate the possibility of raising room rates. You examine what other motels are charging, and from that you produce the following table, relating the price per night and the average number of rooms that are occupied nightly.

P: Price per night	37	45	53
N: Number of rooms occupied	50	40	30

How are the two variables related? What might be an appropriate rate assuming 45 rooms were occupied? 80 rooms? 55 rooms? 10 rooms? Do you think it is reasonable to interpolate and extrapolate in this way? Identify the things that you think are most likely contributing to the change in room rates.

Adapted with permission from Computing and Mathematics: The Impact on Secondary School Curricular, copyright 1984 by the University of Maryland.

RESOURCE CORRELATION

M11:	pp. 165–180
HM11:	pp. 74–85
MPP11:	pp. 151–166
Mathématiques 11:	pp. 130-143, 147–151

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).

Students will be expected to:

- 2.3 use functional notation and graphs to describe functional relationships
  - 2.3.1 solve problems algebraically or by the use and interpretation of graphs that represent functions
- 2.4 determine those relations that are functions
  - 2.4.1 develop and explain tests which could be used to determine if any relation is or is not a function

COMMENTS

The use of functional notation should not be restricted to $f(x)$ but students should be aware that any function can be written in functional notation. For instance area of a circle is a function of the radius or that the area of the radius or that the area of a circle depends on the length of the radius: so $A = \pi r^2$ could be written as $A(r) = \pi r^2$. In this way students do not think that only "mathematics functions" i.e., those labelled as $f(x)$ are the only expression that can be referred to as functions.

In determining which relations are functions, it is important for students to be aware that a function is a special relation - students should have the opportunity to describe the differences between functions and relations, in their own words. Have students give you an example of a function and of a relation and then have them describe the difference, in their own words. This leads to the development of the "Vertical Line Test" - so that students make the connections in their own minds. Students will devise their own "test" while examining the graphs relations and functions.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity 2.4: In determining which relations are functions, give students a number of graphs that are labelled as functions or relations and have them examine the graphs to determine the differences.

Note: This is a good group activity - particularly when grouping the relations and functions - good discussions should happen.

Problem: The amount of timber in a tree grows at a rate that can be predicted from the age and species of a tree; if a species rate is x for the first three years and 1000 trees are planted one year, 2000 the next, and 1500 the third, how much timber will there be at the beginning of the fourth year?

Answer: $1000x^3 + 2000x^2 + 1500x + p$, where p is the amount started at the beginning of the fourth year.

Reprinted from "Applications in Elementary Algebra and Geometry," Applications in School Mathematics, copyright 1979 by the National Council of Teachers of Mathematics.

Problem: What relationship should the dimensions x and y of a rectangle satisfy so that its perimeter and area are numerically equal?

Express one dimension as a function of the other, and note any restrictions on the domain and range. Graph the relation.

Answer: One solution:

From $$2(x + y) = xy$$

$$2x + 2y = xy$$

$$2x = xy - 2y$$

$$2x = y(x - 2)$$

and therefore $$y = {}^{2x}/(x - 2)$$

To satisfy the function, $x \neq 2$ and to satisfy the physical characteristics of the problem (dimensions are positive), x cannot be less than 2. Therefore, $x > 2$ only,

Consider a similar question, "What relationship should exist between the radius r and the altitude h of a right circular cylinder to obtain the numerical equality of the total surface area ($2\pi r^2 + 2\pi rh$) and volume ($\pi r^2 h$)?

Problem: Define the function $f(\frac{a}{b}) = \frac{a+2b}{a+b}$

For example, $f(\frac{1}{2}) = \frac{5}{3}$ $f(\frac{5}{3}) = \frac{11}{8}$ $f(\frac{11}{8}) = \frac{27}{19}$

What happens to successive terms in this sequence ? Check systematically by using a table to keep track of a, b, and $f({}^{a}/_{b})$.

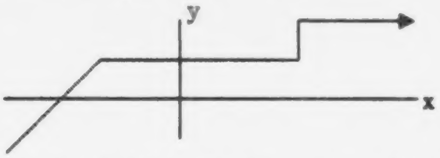
Answer: The successive terms approach $\sqrt{2}$.

Problem 2.3.1: The triangular numbers are 1, 3, 6, 10, 15, 21, 28, . . . Write a formula for the *n*th triangular number as a function of *n*.

Answer: $f(n) = \frac{n(n+1)}{2}$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, April 1987.

Activity 2.4.1: Given a graph such as:



Have students describe whether or not this relation is a function and identify how they made their decision.
(Thanks to G. Stepney, Austin O'Brien H.S.)

TECHNOLOGY INTEGRATION

See HM11: pp. 214, "Computer Application: Testing for a Function."

ELECTIVE SUGGESTIONS

Problem: (An extension of the above problem):
Investigate the function:

$f(a/b) = (a + nb)/(a + b)$ for a natural number n.

Answer: The successive terms approach $\sqrt{n}$.

Problem 2.3.1: Let $f(n + 1) = 5f(n) - 6f(n - 1)$ for a function f defined for all positive integers n. Show that $f(n) = 3^n - 2^n$ is a solution of this equation.

Answer (One solution):
Given $f(n) = 3^n - 2^n$
then $f(n + 1) = 3^{n+1} - 2^{n+1}$
and

$$\begin{aligned} 5f(n) - 6f(n-1) &= 5(3^n - 2^n) - 6(3^{n-1} - 2^{n-1}) \\ &= 5(3\cdot 3^{n-1} - 2\cdot 2^{n-1}) - 6(3^{n-1} - 2^{n-1}) \\ &= 15\cdot 3^{n-1} - 10\cdot 2^{n-1} - 6\cdot 3^{n-1} + 6\cdot 2^{n-1} \\ &= 9\cdot 3^{n-1} - 4\cdot 2^{n-1} \\ &= 3^2\cdot 3^{n-1} - 2^2\cdot 2^{n-1} \\ &= 3^{n+1} - 2^{n+1} \end{aligned}$$

Therefore,
$f(n) = 3^n - 2^n$ is a solution to the equation
$f(n + 1) = 5f(n) - 6f(n - 1)$

RESOURCE CORRELATION

M11:	pp. 160–164
HM11:	pp. 200–203
MPP11:	pp. 50, 257–262
Mathématiques 11:	pp. 210–212, 220–222

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).

Students will be expected to:

- 2.5 illustrate and recognize different kinds of functions algebraically and graphically from the following list: linear functions (including identity and constant functions), polynomial functions (including quadratic and cubic functions), reciprocal functions, absolute value functions, and exponential functions
  - 2.5.1 draw and analyze the graphs of functions using calculators or computers
- 2.6 identify the zeroes of a function as the x-intercepts of its graph
  - 2.6.1 find the zeroes of a function by analyzing its graph and its value for various replacements of the independent variable, using calculators or computers

COMMENTS

It is important for students to have the opportunity to graph all of these functions and to note the differences and similarities between the functions. This task would be very tedious without the use of a graphing software program for the computer or without a graphing calculator.

It would be appropriate to discuss asymptotes while studying the graphs of these functions.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity : Imagine a piece of string tied together in a 50cm loop and formed into a rectangle. Then sketch the three graphs showing the relationship between all possible values of the length and the corresponding width, perimeter, and area. (Find the corresponding algebraic functions defined over the domain 0 through 25.)

Answer: $w(x) = 25 - x$; $p(x) = 50$; $a(x) = 25x - x^2$

Adapted from Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies (2nd ed.), copyright 1988 by Prentice Hall.

Activity 2.5: Once students have had an opportunity to graph the functions and discuss the similarities and differences – give students a number of different graphs and ask them to label the graphs as being the graphs of linear, reciprocal, absolute value, exponential, or polynomial functions. This activity will give you an idea of how well students can recognize the graphs of these functions. This same activity could be done again, given the equations of these functions and having students identify the type of function, given its equation.

Problem 2.5: Suppose the epidemiologists at the National Centers for Disease Control are trying to predict the rate of spread of an apparently incurable, fatal disease. Based on the known number of cases in January 1982 and January 1983, the model they propose is $D(t) = 50e^{.005t}$, which gives the number of thousands of people ill t years after 1982.

a) What can be inferred from the parameters 50 and .005?
b) What is D(20)? What does it represent?
c) What reasons are there for confidence in or doubts about the predictions of this model?

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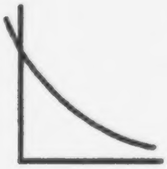
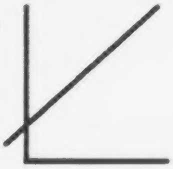
Problem 2.5: Given below are three graphs expressing the relations between pairs of quantitative variables. Study the shapes of these graphs and then answer the questions that follow.

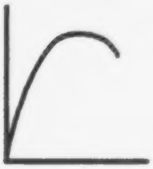
A.



B.



C.



a) Which of the following rules best fits graph A? graph B? graph C?
   (1) $f(x) = 3x + 250$
   (2) $g(x) = -1x^2 + 8x$
   (3) $h(x) = 4e^{.5x}$
   (4) $j(x) = x^2 - 3x$
   (5) $k(x) = 12e^{-x}$

b) Which graph best describes each of the following situations?
   (1) Radioactive decay
   (2) The fuel efficiency of a car as its speed increases
   (3) The costs of running a pizza business as its sales increase

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TECHNOLOGY INTEGRATION

See "A Computer-graphing-based Approach to Solving Inequalities," Mathematics Teacher, May 1989, for an example of using a graphing calculator or a computer program such as Master Grapher to clarify and help solve quadratic and cubic polynomial functions representing surface area, and volume.

ELECTIVE SUGGESTIONS

Problem: If a and b are integers such that $h(x) = x^2 - x - 1$ is a factor of $f(x) = ax^3 + bx^2 + 1$, find b.

Answer: $b = -2$

One solution strategy:
If $h(x)$ is a factor of $f(x)$ then $g(x)$ exists so that $h(x) \cdot g(x) = f(x)$
where $g(x) = cx + d$
so: $(x^2 - x - 1)(cx + d) = ax^3 + bx^2 + 1$
$cx^3 + dx^2 - cx^2 - dx - cx - d = ax^3 + bx^2 + 1$
now $-d = 1$ and $-dx - cx = 0x$ and $dx^2 - cx^2 = bx^2$
so $d = -1$ $\quad -(-1)x - cx = 0x$ $\quad -1x^2 - 1x^2 = bx^2$
$-cx = -1x$ $\quad \therefore -2 = b$
$c = 1$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, June 1988.

RESOURCE CORRELATION

M11:	pp. 142–190, 234–239
HM11:	pp. 200–213, 219, 194–195, 200–202, 219
MPP11:	pp. 275-279, 294-297, 351; 272-274 (extension)
Mathématiques 11:	pp. 201–212, 220–222

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that, for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).

Students will be expected to:

2.7 write, and sketch the graphs of, the inverses of relations and functions

COMMENTS

Using the linear, polynomial, reciprocal, absolute value, and exponential functions students should be able to write the inverse of the equation, of the set of ordered pairs, of the rule and sketch the graph of the inverse. Students should examine the following in studying the inverse of the functions:

1. Are the domain and range different?
2. Is the inverse of a function still a function?
3. What line does the function appear to reflect about when graphing an inverse?
4. Is this the same line for all functions?

In this concept, simple relations could be limited to circles and ellipses or relations expressed as sets of ordered pairs and rules. The questions asked regarding the function could be asked for relations as well and then a comparison between the inverses of relations and functions would be appropriate.

ELECTIVE SUGGESTIONS

Activity: Notice that the rectangle and cylinder problems posed under 2.3.1 (page 69) are of the form $f^{-1}(x) = f(x)$. Such functions are known as involutions.

RESOURCE CORRELATION

M11:	pp. 259–265
HM11:	pp. 198–199, 204–205, 234
MPP11:	pp. 254–256, 262
Mathématiques 11:	pp. 227–233

CONCEPT 3

Students will be expected to demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretchings of the graph of any function.

Students will be expected to:

- 3.1 describe the transformation effects on the graph of $y = f(x)$ of the parameters a and b in $y = f(x-a) + b$
  - 3.1.1 perform an investigation to determine the effects of the parameters a and b on the graph of $y = f(x-a) + b$
- 3.2 describe the transformation effect on the graph of $y = f(x)$ of the parameter c in $y = cf(x)$
  - 3.2.1 perform an investigation to determine the effect of the parameter c on the graph of $y = cf(x)$
- 3.3 describe and sketch the graphs of $y = cf(x-a) + b$ by applying the transformation effects of a, b and c on the graph of $y = f(x)$
  - 3.3.1 predict the graphs of functions written in the form $y = cf(x-a) + b$ given the graph of $y = f(x)$, and verify using calculators or computers

COMMENTS

This concept includes studying $-f(x)$.

Using the linear, polynomial, reciprocal, absolute value, and exponential functions students should be able to predict, given the equation, what the graph of the function will look like. Alternately, students should be able to predict the equation of the function, given its graph.

The use of a computer program or graphing calculators would be useful here and make the investigation of the parameters go very smoothly.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Students should be given an opportunity to graph several functions and identify the effect of the parameters on their own. Have students express this effect in their own words. Once they have had ample opportunity to draw the graphs and examine the relationships, students could be given the graphs and asked to predict the equations of the functions.

TECHNOLOGY INTEGRATION

See "Relating Graphs to Their Equations with a Microcomputer," in Activities for Implementing Curricular Themes from the "Agenda for Action" for a BASIC language program listing and a set of activities designed to encourage students to graphically explore the effects of the constants A, B, and C in the general equation, $y = A|x + B| + C$.

Software:

- Master Grapher (Addison-Wesley)
- Computer Graphing Experiments II (Addison-Wesley)
- Green Globs and Graphing Equations (Sunburst)

ELECTIVE SUGGESTIONS

See "A Simple Periodic Function," Mathematics Teacher, November 1986, for an example and a discussion of a periodic function, complete with graphs of $f(t)$, $A + f(t)$, $B \cdot f(t)$, $f(t + c)$ and $f(Dt)$, which helps to introduce the idea of periodicity without the use of trigonometric functions (to be dealt with in Math 30).

RESOURCE CORRELATION

M11:	pp. 240–258
HM11 :	pp. 206–213
MPP11:	pp. 263–271 (except $f(bx)$)
Mathématiques 11:	pp. 213–216, 219

QUADRATIC FUNCTIONS
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that any function that can be written in the form $y-ax^2+bx+c$, $a \neq 0$, is a quadratic function and that individual quadratic functions have unique characteristics and graphs.	Students will be expected to sketch the graphs of quadratic functions written in standard form, $y=a(x-h)^2+k$.	Students will be expected to investigate the effects of the parameters a, h and k in $y=a(x-h)^2+k$ using a calculator or computer.
	Students will be expected to transform quadratic functions from the general form $y=ax^2+bx+c$ to the standard form $y=a(x-h)^2+k$ by completing the square.	
	Students will be expected to find the vertex, axis of symmetry, domain, range, maximum or minimum values and x- and y-intercepts of a quadratic function from its equation or from its graph.	Students will be expected to solve problems that involve quadratic functions, by analyzing the functions depicted in graphical and equation form.

CONCEPT 1

Students will be expected to demonstrate an understanding that any function that can be written in the form $y = ax^2 + bx + c$, $a \neq 0$, is a quadratic function and that individual quadratic functions have unique characteristics and graphs.

Students will be expected to:

1.1 sketch the graphs of quadratic functions written in standard form, $y = a(x-h)^2 + k$
  1.1.1 investigate the effects of the parameters a, h and k in $y = a(x-h)^2 + k$ using a calculator or computer
1.2 transform quadratic functions from the general form $y = ax^2 + bx + c$ to the standard form $y = a(x-h)^2 + k$ by completing the square

COMMENTS

If you have done the "Functions and Relations" unit, then expectation 1.1 and 1.1.1 will be a review for many students - 1.1 and 1.1.1 helps to make the connection between the two units for students.

You may wish to review the perfect square trinomial (multiplying and factoring) prior to the discussion of transforming the general form of the quadratic to the standard form. It is important that students recognize that although the form of the equation is changing, the graph of the function does not change.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Given:

A	B
$y = \frac{3}{5}x^2$	$y = 7x^2$
$y = -9x^2$	$y = -9(x-2)^2$
$y = \frac{1}{2}x^2$	$y = \frac{1}{2}(x+4)^2 + 5$

Have students describe, in their own words, the changes that occur in the graph of the function as it moves from position A to position B.

(Thanks to S. Kizior, Westwood Community H.S.)

TECHNOLOGY INTEGRATION

The use of a graphing software program or graphing calculators work well with investigating the graphs of quadratic functions.

See "Microcomputer Unit: Graphing Parabolas," Mathematics Teacher, December 1986 for BASIC program listing and a set of activities designed to help students examine the effects of A, B, and C in the general equation, $y = Ax^2 + Bx + C$, and to examine the relationship between the discriminant and the graph of the parabola.

Software:

- Master Grapher (Addison-Wesley).
- Computer Graphing Experiments (Addison-Wesley).
- Green Globs and Graphing Equations (Sunburst).

ELECTIVE SUGGESTIONS

Determine the equation of a parabola which has x-intercepts of 9 and 3, and y-intercept of 18.

One solution:

$$y = a(x-p)^2 + q$$

x int 9, 3 $\quad 0 = a(9-p)^2 + q \Rightarrow q = -a(9-p)^2$

$$0 + a(3-p)^2 + q \Rightarrow q = -a(3-p)^2$$
$$(9-p)^2 = (3-p)^2$$
$$81 - 18p + p^2 = 9 - 6p + p^2$$
$$72 = 12p$$
$$p = 6$$

$$\therefore y = a(x-6)^2 + q$$
$$0 = a(9-6)^2 + q$$
$$0 = 9a + q$$
$$q = -9a$$

y int: 18

$$x = 0$$
$$y = 18$$
$$18 = a(0-6)^2 + q$$
$$18 = 36a + q$$
$$18 = 36a - 9a$$
$$18 = 27a$$
$$a = \tfrac{2}{3}$$
$$q = -9(\tfrac{2}{3}) = -6$$

$$\therefore y = \tfrac{2}{3}(x-6)^2 - 6$$

(Thanks to F. Heinen, Lethbridge Collegiate Institute)

RESOURCE CORRELATION

M11:	pp. 194–216
HM11:	pp. 38–39, 218–228
MPP11:	pp. 293–314
Mathématiques 11:	pp. 238–249

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that any function that can be written in the form $y = ax^2 + bx + c$, $a \neq 0$, is a quadratic function and that individual quadratic functions have unique characteristics and graphs.

Students will be expected to:

1.3 find the vertex, axis of symmetry, domain, range, maximum or minimum values and x- and y-intercepts of a quadratic function from its equation or from its graph
    1.3.1 solve problems that involve quadratic functions, by analyzing the functions depicted in graphical and equation form

COMMENTS

It is important for students to identify the vertex, axis of symmetry, domain, range, maximum or minimum values and x- and y-intercepts from graphs of quadratic functions prior to identifying them from equations. Having students sketching the graphs of the functions reinforces the idea that the equation does represent "something".

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: A toy rocket is fired straight up at an initial velocity of 30m/s. The formula $h = 30t - 4.9t^2$ gives the height of the rocket above the ground t seconds after it is fired. How high does the rocket go? How long is it in the air?

TECHNOLOGY INTEGRATION

See HM11: pp. 236, "Computer Application – Extrema". See MPP11: pp. 327, "Computer Insight: Maxima and Minima"

The Interactive BASIC computer "Root Search" can be used to approximate the maximum height reached.

```
10          INPUT FIRST, LAST, CHANGE

20          PRINT "T", "H"

30          PRINT "__", "__"

40          FOR  T = FIRST TO LAST STEP CHANGE

50          LET H = 30 * T - 4, 9 * T ^2

60          PRINT T, H

70          NEXT T

80          END
```

Input the first and last value of T to be used, and the increment. To refine the approximation of the maximum height, modify the values for FIRST, LAST and CHANGE.

The total travel time of the rocket is found by considering the equation $30t - 4.9t^2 = 0$. This may be solved by factoring. The program may be used by approximating the time T at which H becomes 0. Focus on the domain in which H passes from positive to negative.

Adapted from "Computer Lessons in Algebra," The Ideas of Algebra, K-12, 1988 Yearbook of NCTM.

RESOURCE CORRELATION

M11:	pp. 217–229
HM11:	pp. 229–233
MPP11:	pp. 315–327
Mathématiques 11:	pp. 250–265, 272–279

QUADRATIC EQUATIONS
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.	Students will be expected to use the quadratic formula to solve quadratic equations. Students will be expected to solve equations that contain radicals and rational expressions. Students will be expected to evaluate the discriminant of a quadratic equation and state the nature of its roots. Students will be expected to recognize and explain the relationship between the graphs of quadratic functions and the real or non-real nature of the roots of their corresponding equations.	Students will be expected to solve problems that involve quadratic equations. Students will be expected to solve problems that involve equations which contain radicals and rational expressions.

CONCEPT 1

Students will be expected to demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.

Students will be expected to:

- 1.1 **use the quadratic formula to solve quadratic equations**
  - 1.1.1 **solve problems that involve quadratic equations**

COMMENTS

Although the expectation does not specify that students be involved in developing the quadratic formula, it is advisable that the quadratic formula is derived in the classroom setting, particularly to indicate that the formula was not drawn up out of thin air. You may wish to review completing the square prior to developing the quadratic formula. A review of factoring would be appropriate prior to studying the quadratic formula.

It is also important for students to recognize the relationship between the quadratic formula and factoring. 1.1.1 could have students solving problems that involve factoring or using the quadratic formula to solve quadratic equations. Students should not be limited to using the quadratic formula to solve quadratic equations.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem 1.1: Take two numbers whose sum is one. Which do you think is larger: the square of the smaller added to the larger or the square of the larger added to the smaller? Why?

Answer: They are the same.
(Note that $x^2 + (1 - x) = (1 - x)^2 + x$.)

Reprinted from delta-K, April 1989, p. 52.

Activity 1.1: As students study the quadratic formula, encourage them to continue to view quadratic equations as second-degree polynomials (i.e., the more general case). One way this may be done is to continue to provide equations which may be solved by factoring, as well as with the case of the quadratic formula.

Problem 1.1.1: Find the difference between the larger root and the smaller root of $x^2 - px + (p^2 - 1)/4 = 0$

Answer: 1

Encourage students to consider various possible solution approaches, for example the quadratic formula, the factoring, and the sum and product of the roots.

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TECHNOLOGY INTEGRATION

The use of a graphing software program or graphing calculators help students with investigating the graphs of quadratic functions.

See MP11: p.95, "Computer Insight: Factoring"

See "Microcomputer Unit: Graphing Parabolas," Mathematics Teacher, December 1986 for BASIC program listing and a set of activities designed to help students examine the effects of A, B, and C in the general equation, $y = Ax^2 + Bx + C$, and to examine the relationship between the discriminant and the graph of the parabola.

Software:

- Factoring Algebraic Expressions (Mindscape) - review.

ELECTIVE SUGGESTIONS

Activity: Students can now use: if $x - c$ is a factor of a trinomial function $p(x)$ if (and only if) c is a root of $p(x) = 0$. This fact also implies that when a/b is a root of $f(x) = 0$, then $bx - a$ is a factor. The following approach helps strengthen the connection between factoring and using the quadratic formula.

Consider the following example: Factor the expression $30x^2 + 49x - 98$. Solve the equation $30x^2 + 49x - 98 = 0$ to determine the factors of the given expression.

Then $x = \dfrac{-49 \pm \sqrt{14\,161}}{60} = \dfrac{-49 \pm 119}{60}$

By simplifying for x and changing the equation to an expression equal to zero,

$$\begin{array}{lcl} x = -14/5, & \text{or} & x = 7/6, \\ 5x = -14, & & 6x = 7, \\ 5x + 14 = 0 & & 6x - 7 = 0 \end{array}$$

Then

$$30x^2 + 49x - 98 = (5x + 14)(6x - 7).$$

Students could use this approach to factoring $5x^2 - 8x + 13$ and $x^2 + 4x + 1$. This reinforces the relationship between solving a quadratic equation and factoring.

Extension: The study of quadratic inequalities would be an appropriate extension.

Problem: Find the factors of $36x^2 - 41x - 51$.

Answer: $(9x - 17)(4x + 3)$

RESOURCE CORRELATION

M11:	pp. 119–128
HM11:	pp. 240–248
MPP11:	pp. 332–341
Mathématiques 11:	pp. 296–307

CONCEPT 1 (continued)

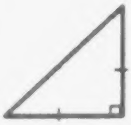
Students will be expected to demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.

Students will be expected to:

1.2 solve equations that contain radicals and rational expressions
    1.2.1 solve problems that involve equations which contain radicals and rational expressions

PROCESS/PROBLEM-SOLVING CONTEXT

Problem 1.2: If the perimeter of an isosceles right triangle is 2m, what is its area in terms of m?



Answer: $A = \dfrac{m^2}{3 + 2\sqrt{2}}$
$= m^2(3 - 2\sqrt{2})$

Reprinted from Mathematics Teacher, National Council of Teachers of Mathematics, May 1987.

Problem 1.2: Solve the equation:
$(x - 2)(x - 3) = 1$

Answer: $x = (5 \pm \sqrt{5})/2$

Reprinted with permission from delta-K, April 1989, "Mathematics for Gifted Students," p. 45.

Problem 1.2: Solve the equation:
$1/(x - 2) + 1/(x - 3) = 1$

Answer: $x = (7 \pm \sqrt{5})/2$

Reprinted with permission from delta-K, April 1989, "Mathematics for Gifted Students," p. 45.

Problem 1.2: Given that the roots of $ax^2 + bx + c = 0$ are m and n, find the condition of a, b, and c so that
a) $m = 1/n$
b) $m = -n$

Answer: a) $a = c$; b) $b = 0$

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For more **Problems** see Problem Solving in Mathematics: Focus for the Future, p. 69, #25; p. 68, #16; p. 66, #14; p. 62, #8.

TECHNOLOGY INTEGRATION

See HM11: p. 62, "Computer Application: the Golden Ratio".

ELECTIVE SUGGESTIONS

See "Existence of Real Roots of a Radical Equation" Mathematics Teacher, May 1987, for a description of solving equations of the form $x - 5\sqrt{x - 4} - 28 = 0$ by transforming them to quadratic equations (for example, $y^2 - 5y - 24 = 0$, where $y = \sqrt{x - 4}$)

RESOURCE CORRELATION

M11:	pp. 106–113, 136–138
HM11:	pp. 246–248, 257–259
MPP11:	pp. 342–348
Mathématiques 11:	pp. 284–295

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.

Students will be expected to:

1.3 evaluate the discriminant of a quadratic equation and state the nature of its roots
1.4 recognize and explain the relationship between the graphs of quadratic functions and the real or non-real nature of the roots of their corresponding equations.

COMMENTS

Although students studied $\sqrt{-a}$ in Mathematics 10, it is not necessary for students to find an approximate value for the complex zeroes. It is important for students to recognize that the quadratic function with complex zeroes does not cross the x-axis. Students should have the opportunity to experiment with various combinations of the zeroes. For instance, can a quadratic exist that has one real and one complex zero? Can a quadratic have two equal complex zeroes?

PROCESS/PROBLEM-SOLVING CONTEXT

Problem 1.3: What are the values of k for which the equation $2x^2 - kx + x + 8 = 0$ will have real and equal roots?

Answer: $k = 9$ or $k = -7$

One solution: Use the quadratic formula to arrive at

$$x = [(k-1) \pm \sqrt{(1-k)^2 - 64}\,]/_4$$

(or simply determine the discriminant as $\sqrt{(1-k)} - 64$.)

Therefore, for the roots to be real and equal,

$$(1-k)^2 = 64$$

$$1 - k = \pm 8$$

Thus, $k = 9$

or $k = -7$

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Problem 1.4: The equation $ax^2 - 2x\sqrt{2} + c = 0$ has real constants a and c with a zero discriminant. Describe the roots.

Answer: The roots are real and equal.

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Problem 1.3: Find the value of the constant k so that the equation $kx^2 + 6x - 4 = 0$ has two equal roots.

Answer: $k = -9/4$

Reprinted from delta-K, April 1989, p. 52

Problem: How can you decide, without a graph, if $y = 5x^2 - 2x + 8$ crosses the x-axis?
(Thanks to G. Stepney, Austin O'Brien H.S.)

Problem: Show that $(1 - \sqrt{2})$ is a root of the equation $x^2 - 2x - 1 = 0$. Describe the process you have used.
(Thanks to G. Stepney, Austin O'Brien H.S.)

TECHNOLOGY INTEGRATION

See M11: pp. 141a–141b, "Computer Power"
See HM11: p. 260, "Computer Application – Factors of a Trinomial"
See MPP11: p. 361, "Computer Insight: Roots of Quadratics"

See "Microcomputer Unit: Graphing Parabolas," Mathematics Teacher, December 1986 for BASIC program listing and a set of activities designed to help students examine the effects of A, B, and C in the general equation, $y = Ax^2 + Bx + C$, and to examine the relationship between the discriminant and the graph of the parabola.

Software:

- Green Globs and Graphing Equations (Sunburst).
- Master Grapher (Addison-Wesley).
- Computer Graphing Experiments (Addison-Wesley).

ELECTIVE SUGGESTIONS

Extension 1.4: Find approximate values for the complex zeroes.

RESOURCE CORRELATION

M11:	pp. 129–133
HM11:	pp. 249–250
MPP11:	pp. 349–351, 357–359
Mathématiques 11:	pp. 308–312

GEOMETRY
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that the perpendicular bisector of a chord passes through the centre of the circle, and that this relationship can be expressed and verified in different ways.	Students will be expected to demonstrate by construction or by computer simulation that the perpendicular bisector of a chord passes through the centre of the circle.	
	Students will be expected to verify the relationship amongst chords, their perpendicular bisectors and the centres of circles using analytic geometry.	Students will be expected to solve problems which involve the relationship amongst chords, the perpendicular bisectors and the centres of circles.
		Students will be expected to calculate the length of a chord, the distance from a chord to the centre of the circle and the radius of the circle given any two of the measurements.
	Students will be expected to provide a logical argument to support the relationship amongst chords, their perpendicular bisectors and the centres of circles.	
Students will be expected to demonstrate an understanding that if a line is tangent to a circle, then it is perpendicular to a radius drawn to the point of contact, and that this relationship can be expressed and verified in different ways.	Students will be expected to demonstrate by construction or by computer simulation that a perpendicular drawn from the point of contact of a tangent passes through the centre of the circle.	
	Students will be expected to verify the relationship between a tangent and the radius drawn to the point of contact, using analytic geometry.	Students will be expected to solve problems which deal with the relationship between a tangent and the radius drawn to the point of contact.
		Students will be expected to calculate, given a point in the exterior of a circle, the length of tangent segments from the point, the lengths of radii to the points of contact of the tangents, and the distance from the point to the centre of the circle, given any two of the measurements.
	Students will be expected to provide a logical argument to support the relationship between a tangent and the radius drawn to the point of contact.	

CONCEPT 1

Students will be expected to demonstrate an understanding that the perpendicular bisector of a chord passes through the centre of the circle, and that this relationship can be expressed and verified in different ways.

Students will be expected to:

- 1.1 demonstrate by construction or by computer simulation that the perpendicular bisector of a chord passes through the centre of the circle
- 1.2 verify the relationship amongst chords, their perpendicular bisectors and the centres of circles using analytic geometry
  - 1.2.1 solve problems which involve the relationship amongst chords, their perpendicular bisectors and the centres of circles
  - 1.2.2 calculate the length of a chord, the distance from a chord to the centre of the circle and the radius of the circle given any two of the measurements
- 1.3 provide a logical argument to support the relationship amongst chords, their perpendicular bisectors and the centres of circles

COMMENTS

This unit is designed to strengthen students' understanding of the connections between inductive, analytic and logical or "deductive" geometry. This will help students in their quadratic relation work in Mathematics 30 and will make the link from coordinate geometry in Mathematics 10 to quadratic relation in Mathematics 30. Hence, providing a logical argument requires that students "think through" the process of an argument.

In order to succeed at the analytic portion of this unit, you may wish to review the coordinate geometry concepts that students studied in Mathematics 10: midpoint of line; distance between two points; relationship of slopes between parallel, perpendicular, and oblique lines; writing equations of lines. In addition, students may be unfamiliar with the equation of a circle, you may wish to spend some time in introducing the equation of a circle - restricted to a circle at the origin as students study circles away from the origin in Mathematics 30.

The analytic approach will involve solving linear - quadratic systems.

In discussing **1.2.2** you may wish to refer back to similar triangles as students have discussed them in detail in Mathematics 10 - this may also give students an alternative "way" of describing the relationship being discussed.

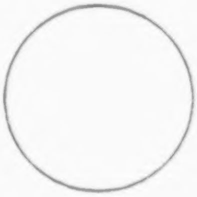
Students may not be familiar with the "inductive" approach in geometry. You may wish to describe the inductive approach as a mathematical experiment. For instance, following a series of experiments, we examine our results, and draw a "conjecture".

(Thanks to C. Felice, St. Marguerite School)

PROCESS/PROBLEM-SOLVING CONTEXT

Activity 1.1: Have students construct several circles of different radii and a chord within each circle. Then have students construct the perpendicular bisector to each chord and have them identify what is happening to the perpendicular bisector in each case. The question becomes: Does the perpendicular bisector of any chord in a circle pass through the centre? (Students could then construct a circle and draw several chords and test their hypothesis.) If the chord is the diameter, does its perpendicular bisector still pass through the centre of the circle? Is there any chord in the circle where the perpendicular bisector does not pass through the centre of the circle? Students could investigate these by constructing circles – it is important that students have the opportunity to express this relationship between the perpendicular bisector of a chord and the centre of a circle in their own words.

Problem 1.1: Given a circle, use a compass and straightedge to determine its centre.



Answer: One possible solution:

Draw two non-parallel chords and construct the perpendicular bisector of each. Their point of intersection is the centre of the circle.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, May 1988.

Problem-Solving Activity 1.2: Building on the previous investigation, students are now aware of the relationship and using their previous knowledge of coordinate geometry can proceed with an investigation of this property in the analytic sense. As a class, discuss the information that might be required to describe this relationship, for instance we might need to know the equation of the circle; the equation of a chord – or the endpoints of the chord; how to find the midpoint of a chord; how to find the equation of a line perpendicular to the chord. Have students work together in groups to verify this relationship analytically given the following:

- the equation of a circle and the equation of a line that goes through the circle
- the equation of a circle and the endpoints of a chord
- the equation of a circle and the midpoint of a chord – can this be done? is there information missing or more information needed?

Have students identify the similarities or differences in solving these – and having students draw out the similarities and

Problem 1.2.2: A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 cm across at the top and 8cm deep. What was the radius of the ball (in centimetres)?
(A) 8 (B) 12 (C) 13 (D) $8\sqrt{3}$ (E) $6\sqrt{6}$

Answer: (C) 13

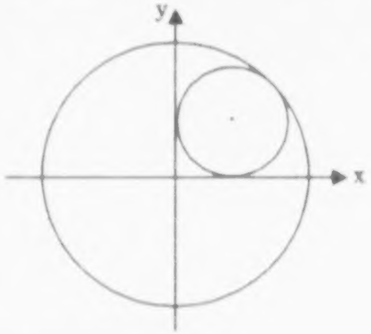
Suggestion: Draw a cross-section of the ball in the ice, and use the Pythagorean Relationship.

Reprinted from Mathematics Teacher. National Council of Teachers' of Mathematics, September 1987.

Activity 1.3: Students must be able to convince either the teacher or a fellow student that this relationship between the perpendicular bisectors of chords and the centres of a circles holds for all circles - and now that they have experienced the analytic approach, the inductive approach, and an introduction to the deductive (through similar triangles in 1.2.2) - they have three approaches to provide this argument in either written or verbal form. The important aspect here is that students are aware that they cannot make a statement without some way to "back it up".

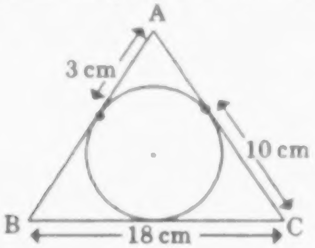
Problems: A circle is inscribed in one quadrant of a circle defined by x2 + y2 = 16 as shown. Determine the radius of the inscribed circle.

$r = 4\sqrt{2} - 4$
≈ 1.66



(Thanks to F. Heinen, Lethbridge Collegiate Institute.)

Problems: Determine the perimeter of $\triangle$ ABC.



(42 cm)

(Thanks to F. Heinen, Lethbridge Collegiate Institute.)

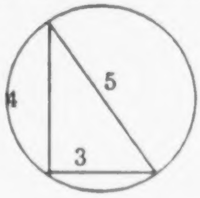
TECHNOLOGY INTEGRATION

See HM11: p. 344, "Computer Application – Circles"

Software:
The Geometric Supposer, Circles (Sunburst).

ELECTIVE SUGGESTIONS

Problem: What is the radius of the circumscribed circle in a 3-4-5 triangle?



Answer: 2.5

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1988.

Problem: Two concentric circles are such that a chord of the larger, 36 cm long, is trisected by the smaller. The sum of the radii of the two circles is also 36 cm. Find the length of the radius of the larger circle.

Answer: 22 cm

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1988.

Problem: Find the value(s) of $(x + y)$ if $x^2 + y^2 = 36$ and $xy = -10$

Answer: $x + y = \pm 4$

One possible approach:

$$\begin{aligned}(x+y)^2 &= x^2 + 2xy + y^2\\ &= x^2 + y^2 + 2xy\\ &= 36 - 20\\ &= 16\\ \therefore x + y &= \pm 4\end{aligned}$$

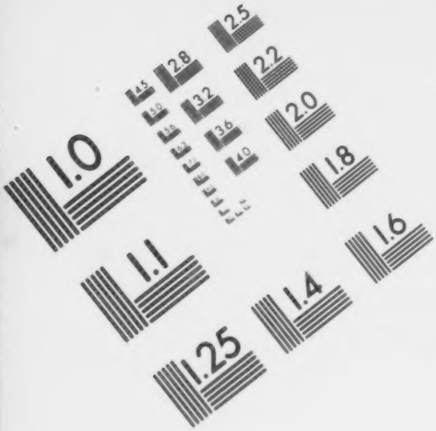
Reprinted from "Flexibility and Algebraic Problem Solving," Mathematics Teacher, April 1987.

Extension: Once the problem has been solved $(x + y = \pm 4)$, plot the four equations: $x^2 + y^2 = 36$; $xy = -10$; $x + y = 4$; $x + y = -4$ (which are functions?) on the same graph. What is observed? Is this to be expected?

Answer:

(a) $x^2 + y^2 = 36$, $y = {}^{-10}/_{x}$ and $y = 4 - x$ intersect at the same points, $(-1.74, 5.74)$ and $(5.74, -1.74)$

(b) $x^2 + y^2 = 36$, $y = {}^{-10}/_{x}$ and $y = -4 - x$ intersect at the same points, $(-5.74, 1.74)$ and $(1.74, -5.74)$



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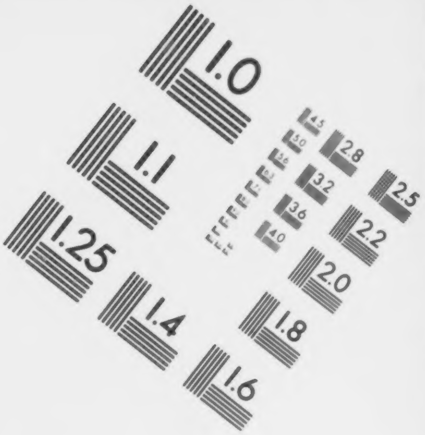

Centimeter
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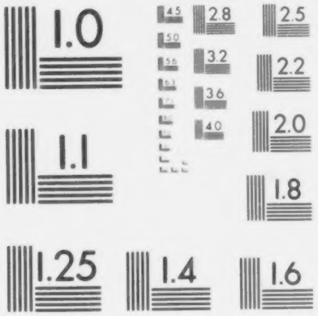
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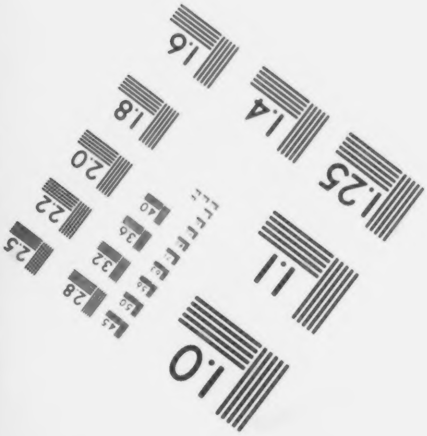

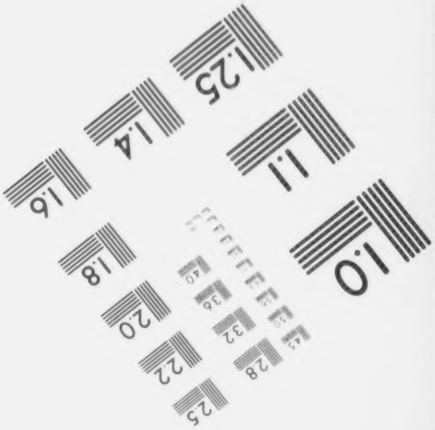

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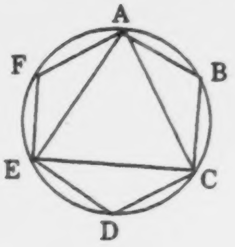
Problem: An equilateral triangle and a regular hexagon are both inscribed in the same circle, as shown. What is the ratio of the area of the hexagon to the area of the triangle?



Answer: 2:1

One solution: Use knowledge of the interior angles of an equilateral triangle and regular hexagon, a chord and its perpendicular bisector, and congruence postulates to demonstrate that the sum of the areas of triangles AEF, ECD, and ACB is equal to that of triangle AEC.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1988.

RESOURCE CORRELATION

M11:	pp. 336–343
HM11:	pp. 323–324, 334–340
MPP11:	pp. 423–426
Mathématiques 11:	pp. 334–348

CONCEPT 2

Students will be expected to demonstrate an understanding that if a line is tangent to a circle, then it is perpendicular to a radius drawn to the point of contact, and that this relationship can be expressed and verified in different ways.

Students will be expected to:

- 2.1 demonstrate by construction or by computer simulation that a perpendicular drawn from the point of contact of a tangent passes through the centre of the circle
- 2.2 verify the relationship between a tangent and the radius drawn to the point of contact, using analytic geometry
  - 2.2.1 solve problems which deal with the relationship between a tangent and the radius drawn to the point of contact
  - 2.2.2 calculate, given a point in the exterior of a circle, the length of tangent segments from the point, the lengths of radii to the points of contact of the tangents, and the distance from the point to the centre of the circle, given any two of the measurements
- 2.3 provide a logical argument to support the relationship between a tangent and the radius drawn to the point of contact

COMMENTS

As in concept 1, it is expected that students will develop an appreciation for the "interconnectedness" between inductive, analytic, and logical geometry. In this light, students have not formally discussed tangent - you may wish to discuss the definition of the tangent - omitting the property that indicates that tangents are perpendicular to the radius at the point of tangency - so students should work from the definition of a tangent to examine the property. Constructing a tangent to a circle becomes a good problem-solving activity - how do you construct a tangent to a circle from a point outside a circle? Can a tangent to a circle be constructed from a point inside the circle?

The analytic approach will involve solving a linear - quadratic system.

In discussing 2.2.2 you may wish to refer back to similar triangles as students have discussed them in detail in Mathematics 10 - this may also give students an alternative "view" of describing the relationship being discussed.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity 2.1: Have students construct several circles of different radii and a tangent to each circle. Then have students draw the radius to each point of tangency and have them measure the angle between the radius and the tangent. The question is: Does the tangent to a circle always meet the radius to the point of tangency at 90°? Does this only happen at one of the points of tangency - or does it happen at both of the tangents that can be drawn from any point outside the circle to the circle? It is important then that students have the opportunity to express this relationship between the perpendicular bisector of a chord and the centre of a circle in their own words.

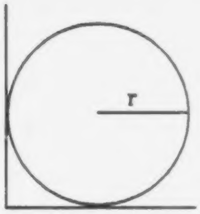
Problem-Solving Activity 2.2: Building on the previous investigation, students are now aware of the relationship – and using their previous knowledge of coordinate geometry can proceed with an investigation of this property in the analytic sense. As a class, discuss the information that might be required to describe this relationship, for instance we might need to know the equation of the circle; the equation of a tangent – or the point of tangency; how to find where the radius meets the circle; how to find the equation of a line perpendicular to the tangent or radius. Have students work together in groups to verify this relationship analytically given the following:

- the equation of a circle and the quotation of a line that is tangent to the circle
- the equation of a circle, the length of a radius, and a point outside of a circle
- the equation of a circle, a point outside a circle and th equation of the radius – can this be done? is the information missing or more information needed?

Have students identify the similarities or differences in solving these questions – having students draw out the similarities and differences would assist in solving other problems. Have students attempt to identify whether this relationship can be described analytically without the equation of the circle? or without the equation of the tangent line?

Activity 2.3: Students must be able to convince either the teacher or a fellow student that this relationship between the tangent and the radius to the point of tangency holds for all circles – they have three approaches to provide this argument in either written or verbal form. The important aspect here is that students are aware that they cannot make a statement without some way to "back it up."

Problem: In the drawing below, a circle of radius r fits in the right-angled corner. What is the radius of the largest circle that will fit in the same corner between the larger circle and the corner?

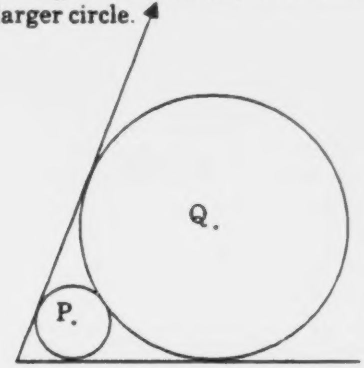


Answer: $r = (3 - 2\sqrt{2})$ or ≈ 0.172

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, April 1988.

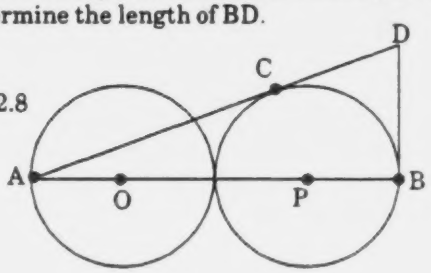
Problem; A circle with centre p and radius 10 is tangent to the sides of an angle of 60°. A larger circle with centre Q is tangent to the sides of the angles and to the first circle. Determine the radius of the larger circle.

Answer: $r = 30$



Problem: Two circles with centres O and P, each with a radius of 2 are tangent to each other. The line OP cuts the circles at A and B. Lines AC and BD are tangent to the circle with centre P and intersect at D. Determine the length of BD.

Answer: $BD = 2\sqrt{2} \approx 2.8$



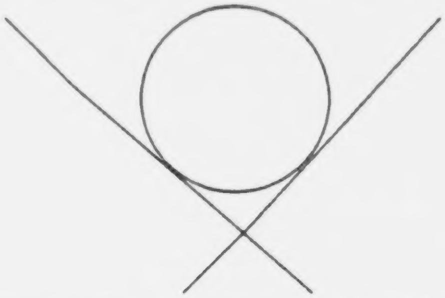
(Thanks to F. Heinen, Lethbridge Collegiate Institute.)

TECHNOLOGY INTEGRATION

Software: The Geometric Supposer, Circles (Sunburst).

ELECTIVE SUGGESTIONS

Problem: Six regions can be formed when two tangents are drawn to a circle. Investigate the maximum number of regions that can be formed if 3, 4, 5, . . . n tangents are drawn.



Answer: In general, for n tangents, $\frac{(n+1)(n+2)}{2}$ regions are formed.

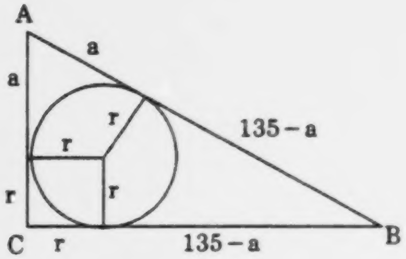
Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, February 1987.

Problem: The perimeter of a right triangle is 324cm and its hypotenuse is 135cm. Find the radius of the circle that can be inscribed in the triangle.

Answer: radius = 27cm

One solution:

The key to this solution as revealed in the sketch is to recognize that tangents to a circle from the same external point are congruent.



Therefore $2a + 2r + 2(135 - a) = 324$

$2r = 54$

$r = 27$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1986.

For a description, with examples, of a procedure for solving quadratic equations ($x^2 - px + q = 0$) using coordinate geometry and the geometry of circles, tangents, secants and chords, see "Circles, Chords, Secants, Tangents and Quadratic Equations," Mathematics Teacher, December 1976. (If acquiring this issue is difficult, contact the author, Dr. A.T. Olson, Department of Secondary Education, University of Alberta, Edmonton, Alberta, T6G 2G5.) (Also strongly ties to Quadratic Equations.)

RESOURCE CORRELATION

M11: pp. 357–362
HM11: pp. 325–327, 338–340
MPP11: pp. 438–440
Mathématiques 11: pp. 349–354

TRIGONOMETRY
PROGRAM EMPHASIS – 12%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that angles can be drawn on a coordinate plane and that their trigonometric ratios can be determined with respect to the coordinates of points on the plane.	Students will be expected to recognize and sketch positive and negative angles in standard position.	
	Students will be expected to determine the reference angle for an angle drawn in standard position on a coordinate plane.	
	Students will be expected to express the sine, cosine and tangent ratios for any angle, θ, in standard position on a coordinate plane in terms of x,y and r, where r is the length of the terminal side and (x,y) is the end point.	
	Students will be expected to determine the sine, cosine and tangent ratios for any angle.	Students will be expected to use a calculator to determine the sine, cosine and tangent ratios of any angle.
	Students will be expected to determine any two values of x,y,r and θ for an angle in standard position given the other two.	
Students will be expected to demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.	Students will be expected to find the measures of sides and angles in diagrams involving multiple right triangles in two or three dimensions.	Students will be expected to solve problems involving multiple right triangles in two or three dimensional.
	Students will be expected to find the measure of unknown sides and angles in oblique triangles by applying the sine law.	Students will be expected to verify the sine law. Students will be expected to solve problems, including the ambiguous case, by using the sine law.
	Students will be expected to find the measure of unknown sides and angles in oblique triangles by applying the cosine law.	Students will be expected to verify the cosine law. Students will be expected to solve problems by using the cosine law. Students will be expected to recognize that the cosine law is the generalization of the Pythagorean theorem.

CONCEPT 1

Students will be expected to **demonstrate an understanding that angles can be drawn on a coordinate plane and that their trigonometric ratios can be determined with respect to the coordinates of points on the plane.**

Students will be expected to:

1.1 **recognize and sketch positive and negative angles in standard position**

1.2 **determine the reference angle for an angle drawn in standard position on a coordinate plane**

COMMENTS

In Math 10, students have studied the three primary trigonometric ratios in relationship to the solution of right triangle problems. It is the intent, in Math 20, to extend the study of trigonometry to oblique triangles in order to prepare students for their study of trigonometry in Math 30 where they examine the trigonometric functions.

Note: The study is restricted to the primary trigonometric ratios so that students gain a complete understanding of sine, cosine and tangent. As students have calculators to find trigonometric ratios, introduction of the reciprocal ratios will be done in Math 30 as the inverses of the sine, cosine and tangent functions. The introduction in Math 30 then reinforces the material dealing with the inverse of functions from Functions and Relations in Math 20.

TECHNOLOGY INTEGRATION

Video:
Trigonometric Functions I (TV Ontario): #1, Trigonometric Ratios (review); #2, Solving Right Triangles (review); #3, Angles on the Plane.

RESOURCE CORRELATION

M11:	pp. 400–408
HM11:	pp. 298–300 (except reciprocal ratios)
MPP11:	pp.366–369 (except reciprocal ratios)
Mathématiques 12:	pp. 142–146

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that angles can be drawn on a coordinate plane and that their trigonometric ratios can be determined with respect to the coordinate of points on the plane.

Students will be expected to:

1.3 express the sine, cosine and tangent ratios for any angle, θ, in standard position on a coordinate plane in terms of x, y, and r, where r is the length of the terminal side and (x, y) is the end point

1.4 determine the sine, cosine and tangent ratios for any angle

  1.4.1 use a calculator to determine the sine, cosine and tangent ratios of any angle

1.5 determine any two values of x, y, r and θ for an angle in standard position given the other two.

COMMENTS

This beginning work in relating trigonometry as a circular function is important for students in transferring to math 30. It is important that students be given the opportunity to make their own generalizations in terms of when the sine of an angle is positive or negative. Understanding what sign (positive or negative) the trigonometric ratio of a particular angle is important particularly when students examine the reasonableness of the result provided by a calculator. Hence, memorization of the "CAST" rule is not important, but understanding the relationships within the rule IS important.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students sketch several angles in standard position on a coordinate plane and determine the trigonometric ratios. Once this has been completed, have students examine the trigonometric ratios they have found to determine a relationship between the quadrant that the terminal arm of the angle lies in and the sign of the trigonometric ratio. This opportunity can lead to students examining the "circular" nature of trigonometry. Students will probably notice that the trigonometric ratios for angles in standard position are the same but that the signs differ - or that the sine ratio for some angle is the same as the cosine ratio for another angle. Examining these relationships will assist students in being able to determine whether or not the answer provided by a calculator is reasonable or not.

Problem: Explain why the following statements are true.
$-1 \leq \sin\theta \leq 1$ and $-1 \leq \cos\theta \leq 1$

(Thanks to M. Ray, J.A. Williams H.S.)

ELECTIVE SUGGESTIONS

Teacher Comment: I have a huge homemade poster of the unit circle up on a bulletin board for Math 30. I was most impressed when a Math 20 student asked why Math 30 students complained about memorizing the unit circle when it was so easy. We then discussed the points on the circle. From this lecture they explained to me how to find other possible angles given one trigonometric ratio.

The day before they noticed the relationship between special triangles and the unit circle, we found the two other trigonometric ratios given one trigonometric ratio and the quadrant. I was showing them or reminding them to check the sides of the triangle to make sure they seemed correct - the hypotenuse was/is actually the longest side. I asked them to make a statement about the sine or cosine ratio. With little effort someone said both $\sin\theta$ and $\cos\theta$ must lie between -1 and 1 inclusive. We then discussed the other ratios. I had the posters of the trigonometric graphs up on the bulletin board and was able to show them the graphs. They looked at them and nodded as if to say "obviously, what did you think they will look like."

I think this group will have a much better understanding of trigonometry (actually they already do) than the Math 30 group.

(Thanks to M. Ray, J.A. Williams H.S.)

RESOURCE CORRELATION

M11:	pp. 409–413
HM11:	pp. 300–303 (except reciprocal ratios)
MPP11:	pp. 370–378 (except reciprocal ratios)
Mathématiques 12:	pp. 146–152

CONCEPT 2

Students will be expected to demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.

Students will be expected to:

2.1 find the measures of sides and angles in diagrams involving multiple right triangles in two or three dimensions

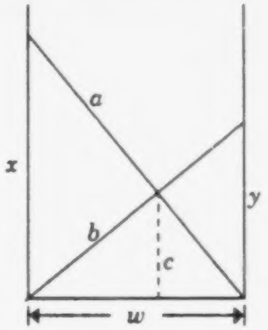
  2.1.1 solve problems involving multiple right triangles in two or three dimensions

COMMENTS

Solving right triangles in two or three dimensions is an extension of the Math 10 program where students solve right triangles in two dimensions.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem 2.1: Two ladders are propped across a narrow alley. The lengths of the ladders are $a = 9\text{m}$ and $b = 6\text{m}$. The alley is $w = 4\text{m}$ wide. Find the height, c, above the ground of the intersecting point of the two ladders.



Answer: $c = 2.87\text{m}$

One solution:
Using trigonometry;

$\cos\theta = {}^{w}/_{b} = {}^{4}/_{6}$

$\therefore \theta = 48.2°$

$\cos\theta = {}^{w}/_{a} = {}^{4}/_{9}$

$\therefore \theta = 63.6°$

Using trigonometry, or Pythagorean Theorem

$y = 4.47\text{m}$

$x = 8.05\text{m}$

By similar triangles:

$^{c}/_{4.47} = {}^{r}/_{4}$

$\therefore 4c = 4.47r$

and

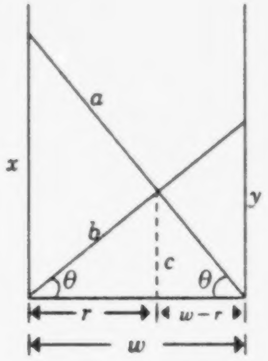
$^{c}/_{8.06} = {}^{4-r}/_{4}$

$\therefore 4c = 8.06\,(4 - r)$

$= 32.24 - 8.06r$

$\therefore 4.47r = 32.24 - 8.06r$

and $r = 2.57$



Substituting,

$4c = 4.47\,(2.57)$

and $c = 2.87\text{m}$

RESOURCE CORRELATION

M11:	pp. 390–393, 414–417
HM11:	pp. 295–297, 303, 311–312
MPP11:	pp.399–401
Mathématiques 12:	pp. 203–213, 234–240

CONCEPT 2

Students will be expected to **demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.**

Students will be expected to:

2.2 find the measures of unknown sides and angles in oblique triangles by applying the sine law
- 2.2.1 verify the sine law
- 2.2.2 solve problems, including the ambiguous case, by using the sine law

COMMENTS

In introducing the Sine Law, it is important for students to see that is developed by comparing triangles - however to have students memorize the development or "proof" is not necessary.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: In a 5-12-13 right triangle, what is the length of the bisector of the larger acute angle?

Answer: $5\sqrt{13}/3 \approx 6.01$

Reprinted from Mathematics Teacher. National Council of Teachers' of Mathematics, March 1988.

TECHNOLOGY INTEGRATION

See M11: pp. 427–429, "Computer Power"
See HM11: p. 314, "Computer Application - Law of Sines"

Video:
Trigonometric Functions I (TV Ontario): #4, The Sine Law; #6, Applications of the Sine Law and the Cosine Law.

RESOURCE CORRELATION

M11:	pp. 425–433
HM11:	pp. 305–307, 313
MPP11:	pp.387–391, 398
Mathématiques 12:	pp. 214–221, 230–234

CONCEPT 2

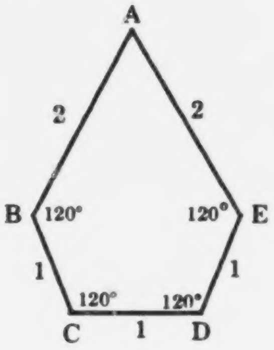
Students will be expected to demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.

Students will be expected to:

2.3 find the measures of unknown sides and angles in oblique triangles by applying the cosine law
  - 2.3.1 verify the cosine law
  - 2.3.2 solve problems by using the cosine law
  - 2.3.3 recognize that the cosine law is the generalization of the Pythagorean Theorem

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: How long is AC?



Answer: √7

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1988.

TECHNOLOGY INTEGRATION

See M11: pp. 427 – 429, "Computer Power"
See MPP11: p. 269, "Computer Insight: Classifying Triangles"

Video:
Trigonometric Functions I (TV Ontario): #5, The Cosine Law; #6, Applications of the Sine Law and Cosine Law.

ELECTIVE SUGGESTIONS

Problem: Find all values of x that satisfy $x^2 - \cos x + 1 = 0$.

Answer: $x = 0$.

$$
\begin{aligned}
& x^2 - \cos x + 1 = 0 \\
& x^2 + 1 = \cos x \\
\text{But } & -1 \leq \cos x \leq 1 \\
\therefore \quad & -1 \leq x^2 + 1 \leq 1 \\
& -2 \leq x^2 \leq 0
\end{aligned}
$$

working with real solutions only, we have $x = 0$

RESOURCE CORRELATION

M11:	pp. 418–424
HM11:	pp. 308–310, 313
MPP11:	pp. 392–398
Mathématiques 12:	pp. 222–229

REVERSE TEXTUAL CORRELATION
Addison-Wesley Publishers Limited
Mathematics 11
(National Edition)

Page	Unit	Learner Expectation Number	TRM Page
1–19	Number Systems		
20–23	Radicals and Exponents	2.1, 2.2	32
24–27	Problem Solving		
28–29	Rationalizing Denominators		
31–32	Number Systems		
33–37	Radicals and Exponents	2.3	32
40–50	Exponent Laws		
51–55	Radicals and Exponents	1.1, 1.2	29
56–57	Problem Solving		
59–63	Solving Equations Involving Exponents		
66–80	Polynomial Operations		
81	Factoring the Sum and Difference of Cubes		
82–84	Polynomial Operations		
85–88	Rational Expressions	1.1, 1.2	39
89–101	Rational Expressions	1.3, 1.4, 1.5	41
106–128	Quadratic Equations	1.1, 1.2	85
129–135	Quadratic Equations	1.3, 1.4	87
136–138	Quadratic Equations	1.1.1, 1.2.1	85
142–149	Functions and Relations	1	61
150–156	Functions and Relations	2.4	68
160–164	Functions and Relations	2.3	68
165–183	Functions and Relations	2.1, 2.2	66
184–185	Problem Solving		
186–190	Extending Direct and Inverse Variation		
194–197	Quadratic Functions	1	
198–213	Quadratic Functions	1.1	79
214–216	Quadratic Functions	1.2	79
217–221	Quadratic Functions	1.3	79
222–229	Quadratic Functions	1.3.1	81
234–239	Functions and Relations	2.5	81
240–258	Functions and Relations	3	71
259–265	Functions and Relations	2.7	75
266–268	Functions and Relations	1	74
269–275	Transforming the Equation of the Circle		61
276–277	Problem Solving		
278–280	Quadratic Inequalities		
284–334	Plane Geometry		
336–376	Circle Geometry		
378–389	Trigonometry Review		
390–393	Trigonometry	2.1	108
394–399	Trigonometric Ratios of Special Angles		
400–413	Trigonometry	1	105-107
414–417	Area of a Triangle		
418–424	Trigonometry	2.3	110
425–433	Trigonometry	2.2	109
434–435	Problem Solving		
438–479	Probability	1, 2, 3	47-58

REVERSE TEXTUAL CORRELATION
Holt, Rinehart and Winston of Canada, Limited
Holtmath 11

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1–8	Polynomial Operations: Review		
9–17	Problem Solving		
18–25	Perimeter, Area and Volume		
32–37	Polynomials Operations		
38–39	Quadratic Functions	1.2	79
40–51	Polynomials Operations		
52–61	Rational Expressions	1.1, 1.2, 1.3, 1.4, 1.5	39–43
66–73	Ratio and Proportion		
74–81	Functions and Relations	2.1	66
82–85	Problem Solving		
92–95	Number Systems		
96–99	Exponent Laws		
100	Radicals and Exponents	1	29
100–111	Radicals and Exponents	2, 1.1, 1.2	29–36
112–116	Exponents: Applications, Exponential Equations		
120–129	Geometry: Review, Parallel Line, Angles, Constructions		
130–132	Geometry: Angles of a Triangle		
133–138	Geometry	1	93
139–143	Geometry: Quadrilaterals, Trapezoids		
144–145	Geometry: Indirect Proof		
150–151	Analytic Geometry		
152	Functions and Relations	2.1, 2.2	66
154–161	Functions and Relations	2.5	71
162–186	Analytic Geometry		
192–195	Functions and Relations	1	61
194–195	Functions and Relations	2.1	66
196–197	Graphing Inequalities		
198–213	Functions and Relations	2.7, 2.5, 2.3, 2.4, 3.1, 3.2, 3.3	66-76
218–228	Quadratic Functions	1.1, 1.2	79
219	Functions and Relations	2.6	71
219–233	Quadratic Inequalities	1.3	81
234	Functions and Relations	2.7	74
235	Quadratic Inequalities		
240–259	Quadratic Equations	1.1, 1.2, 1.3, 1.4	85-88
265–286	Systems of Equations		
288–294	Trigonometry: Review		
295–303	Trigonometry	2.1, 2.1.1, 1.1, 1.3, 1.4, 1.5	105-108
304	Trigonometry: Equations		
305–313	Trigonometry	2.2, 2.3, 2.1	108-111
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331–333	Circle Geometry: Arcs and Sectors		
334–337	Geometry	1	93
338–340	Equations of Circles II		
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341–343	Circle Transformations		
350–354	Probability	1, 2.1, 2.2	47-54
355–356	Probability	3	55
357–384	Statistics		
387–407	Sequences and Series		
408–425	Compound Interest and Annuities		
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REVERSE TEXTUAL CORRELATION
Nelson Canada
Mathematics: Principles and Process 11

Page	Unit	Learner Expectation Number	TRM Page
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21–24	Exponent Laws		
25–40	Polynomials: Operations		
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50	Functions and Relations	2.3	68
56–68	Polynomials Operations		
69–71	Quadratic Equations	1	85
72–77	Polynomial Operations		
78–86	Rational Expressions	1.1, 1.2, 1.3, 1.4	39–43
89–93	Rational Expressions	1.5	41
100–102	Number Systems		
103–104	Laws of Exponents		
105–109	Radicals and Exponents	1.1, 2.2	29–36
110–112	Solving Exponential Equations		
113–135,137	Radicals and Exponents	2	32
141–150	Ratio and Proportion		
151–169	Functions and Relations	2.1, 2.2	66
170–171	Problem Solving		
175–212	Systems of Equations		
214–215	Functions and Relations	2.1, 2.2	66
216–219	Analytic Geometry		
220–221	Functions and Relations	2.1, 2.2	66
222–248	Analytic Geometry		
250–253	Functions and Relations	1	61
254–256	Functions and Relations	2.7	74
257–262	Functions and Relations	2.3, 2.4	68
262	Functions and Relations	2.7	74
263–271	Functions and Relations	3.1, 3.2, 3.3	75
272–282	Functions and Relations	2.5	71
283–288	Operations with Functions		
294–300	Quadratic Functions	1.3	81
296	Functions and Relations	2.6	71
301–325	Quadratic Functions	1.1, 1.2, 1.3.1	79–82
332–351	Quadratic Equations	1.1, 1.2, 1.3, 1.4	85–88
351	Functions and Relations	2.6.1	71
352–354	Quadratic Equations	1.1.1 (extension)	85
352–354	Quadratic Equations	1.2.1 (extension)	87
355–356	Quadratic Inequalities		
357–359	Quadratic Equations	1.4 (extension)	89
366–369	Trigonometry	1.1, 1.2	105
370–378	Trigonometry	1.3, 1.4, 1.5	106
379–381	Trigonometric Equations		
382–386	Trigonometry: Solving Right Triangles: Review		
387–401	Trigonometry	2.2, 2.3, 2.1	108-111
406–408	Geometry: Vocabulary and Concepts		
409–422	Geometry: Constructions and Proofs		
423–426	Geometry with Circles and Chords		
427–431	Circle Geometry: Angles:		
432–437	Circle Geometry: Concyclic Points and Cyclic Quadrilaterals		
438–440	Properties of Tangents: Circles		
441–446	Circle Geometry: Tangents and Chords		
447–448	Circle Geometry: Arcs and Sectors		
454–457	Probability	1	47
458–463	Probability	2.1, 2.2	51-54
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REVERSE TEXTUAL CORRELATION – ÉDITIONS BEAUCHAMIN LTÉE
Mathématiques: 11 (11e Année)

Page	Unit	Learner Expectation Number	TRM Page
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47–49	La revue polynôme		
70–72	Les expressions rationnelles	1.2	39
73–75	Les expressions rationnelles	1.3	39
76–79	Les expressions rationnelles	1.4	41
80–81	Les expressions rationnelles	1.5, 1.5.1	41
83–105	La revue des systèmes numériques		
106–109	Résolution d'équations exponentielles		
114–129	Applications des rapports		
130–143	Fonctions et relations	2.1, 2.2	66
144–146	Applications de la variation combineé		
147–151	Fonctions et relations	2.1, 2.2	66
155–200	La fonction linéaire: résolution de systèms d'équations		
202–204	Fonctions et relations	1.2	61
205–209	Fonctions et relations	2.3, 2.4	68
210–212	Fonctions et relations	2.3	68
213–219	Fonctions et relations	3	75
220–222	Fonctions et relations	2.4	68
223–226	Fonctions particulières		
227–233	Fonctions et relations	2.7	74
238–244	Fonctions quadratiques	1.3.1	81
245–249	Fonctions quadratiques	1.1.1	79
250–252	Fonctions quadratiques	1.3	81
253–256	Fonctions quadratiques	1.2	79
257–265	Fonctions quadratiques	1.1	79
266–268	Réciproque d'une fonction du second degré		
269–271	Regions du second degré		
272–280	Équations quadratiques	1.1, 1.2	85-88
284–302	Équations quadratiques	1.3, 1.4	89
303–307	Équations quadratiques	1.1, 1.2	85–88
308–316	Équations quadratiques	1.3, 1.4	89
319–333	La géométrie analytique		
334–337	La géométrie		
338–344	La géométrie analytique		
345–348	La géométrie	1.2	93
349–357	La géométrie	2.2	98
358–360	Résolution de problèmes		
363–404	Certains aspects des suites et des séries		
405–459	Mathématiques de l'investissement		

MATH 23

Program Emphasis

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises of 80% of the course and contains the concepts, skills, and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to the problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 23

Unit	Suggested Program Emphasis (in %)
Powers and Radicals	10
Algebra	10
Linear Relations	10
Systems of Equations	13
Probability	11
Geometry	13
Trigonometry	13
Elective	20

Attitudes

1. *Students will be expected to* demonstrate an attitude associated with mathematical literacy. In particular, students will be expected to:

    1.1 be confident in their mathematical knowledge and in their ability to acquire new knowledge
    1.2 demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems
    1.3 develop intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge
    1.4 exhibit an attitude of curiosity and spontaneity, and appreciate creativity and innovation in representing situations mathematically
    1.5 be critical and constructive in approaching new ideas and new processes
    1.6 be aware of the importance of communication skills in mathematics
    1.7 appreciate the usefulness of computational competence, mathematical processes and problem-solving skills which are used in the decision-making and modeling processes in our society
    1.8 appreciate the contributions of mathematics to our culture and civilization.

Problem Solving

1. *Students will be expected to* demonstrate an understanding of the variety of procedures that can be used to understand problems. In particular, students will be expected to:

    1.1 read the problem thoroughly
    1.2 identify and clarify key components
    1.3 restate the problem, using familiar terms
    1.4 evaluate the given information as to sufficiency and relevancy
    1.5 interpret pictures, charts and graphs
    1.6 determine hidden assumptions
    1.7 ask relevant questions
    1.8 identify given, needed and wanted information
    1.9 diagram or model the problem situation
    1.10 use suitable notation
    1.11 determine valid inferences
    1.12 simulate a problem situation
    1.13 formulate situations into identifiable problems.

2. *Students will be expected to* develop a variety of strategies for use in the solution of mathematical problems. In particular, students will be expected to:

    2.1 conduct an investigation
    2.2 use estimation and approximation
    2.3 develop equations or use formulae
    2.4 use flow charts
    2.5 make lists and charts
    2.6 look for patterns
    2.7 work backward
    2.8 break the problem into smaller parts
    2.9 look for a simpler or related problem

2.10 make diagrams or models
2.11 use manipulatives
2.12 choose and sequence a series of mathematical operations
2.13 sketch the graph of a problem situation
2.14 establish procedures to gather and organize data
2.15 apply empirical or inductive processes
2.16 use geometric construction and measurement techniques
2.17 make and test a conjecture.

3. *Students will be expected to* develop a variety of skills that can be used to carry out the plan for the solution of a problem. In particular, students will be expected to:

3.1 apply selected strategies
3.2 present ideas clearly
3.3 document the solution process
3.4 use appropriate group behaviours

3.5 use calculators and computers
3.6 evaluate problem-solving strategies for effectiveness
3.7 alter or abandon non-productive strategies
3.8 search for additional information
3.9 ask questions
3.10 be open to inspirations, intuitions and "bright ideas".

4. *Students will be expected to* employ a variety of skills to help them look back over the solution of a problem. In particular, students will be expected to:

4.1 determine the reasonableness of an answer
4.2 explain the solution in oral or written form
4.3 consider the possibility of additional solutions
4.4 search for other strategies and processes of solution
4.5 create and solve similar problems
4.6 note the characteristics that will be identifiable in similar problems
4.7 make a generalization
4.8 examine the assumptions made and simplifications and modifications used for accuracy, effectiveness and efficiency.

POWERS AND RADICALS
PROGRAM EMPHASIS – 10%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding of the relationship between radicals and exponents.	Students will be expected to use the following terms as they occur in the study of radicals: index, radicand, radical sign, root.	
	Students will be expected to transform expressions of the form $x^{a/b}$ to $^{b}\ x^{a}$ or $(^{b}x)^{a}$ and vice versa where x is a rational number and $b = 2$.	
	Students will be expected to calculate approximations for the square and cube roots of rational numbers	Students will be expected to find approximations for square and cube roots of rational numbers by using a calculator.
Students will be expected to demonstrate an understanding of how interest accumulates if it is allowed to compound.	Students will be expected to calculate compound interest by using an interative process.	
	Students will be expected to calculate compound interest using the formula $A = P(1 + i)^{n}$, where A is the amount (principal and interest), i is the interest rate per conversion period and n is the number of periods.	Students will be expected to use a calculator or spreadsheet in the calculation of compound interest. Students will be expected to solve problems involving the calculation of compound interest.

CONCEPT 1

Students will be expected to demonstrate an understanding of the relationship between radicals and exponents.

Students will be expected to:

1.1 use the following terms as they occur in the study of radicals: index, radicand, radical sign, root

1.2. transform expressions of the form $x^{a/b}$ to $\sqrt[b]{x^a}$ or $(\sqrt[b]{x})^a$ and vice versa where x is a rational number and b = 2

1.3 calculate approximations for the square and cube roots of rational numbers

  1.3.1 find approximations for square and cube roots of rational number by using a calculator

COMMENTS

The intent of this concept is for students to understand the relationship between rational exponents and radicals. In Grade 9, students determine the square root of perfect square numbers, in Grade 10 students determine the square root of nonperfect squares. In Mathematics 23 this expectation is extended into the cube root of any rational number. Your students may require a review of the previous work (particularly number systems and exponent laws) prior to introducing the new concept - particularly when discussing the need for rational exponents or demonstrating the importance of rational exponents. This expectation is extending the work in the Real Number system.

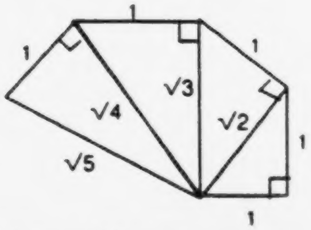
This concept provides an opportunity to demonstrate the proper use of the calculator - students should see that they can use both the y^x key as well as the $\sqrt[x]{\ }$ key when evaluating exponential and radical expressions - the use of the calculator does not limit the value of the base. This is also a good time to reinforce students' estimation skills - i.e. what do you think will be the cube root of 8? - and then evaluate the cube root of 8 on a calculator. More extreme examples could be: What is the value of $6^{7/3}$? Students could estimate the value and then use the calculator to find the value.

Note: The work with operations on radical numbers will be done in Mathematics 33. In addition, it is not expected that students be able to work with the cube root of a variable. Only rational numbers should be used. An extension of the concept could be to include work with a variable. You could include the use of a negative exponent - this would reinforce the laws of exponents.

Definitions for index, radicand, radical sign, and root can be found in the Mathematics Dictionary Fourth Edition.

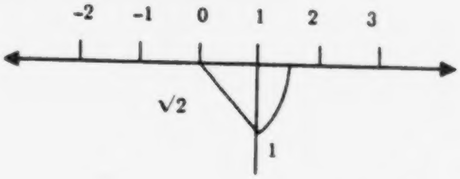
PROCESS/PROBLEM-SOLVING CONTEXT

Activity: 1.2 Have students carefully construct the following right triangles and measure the hypotenuse in each triangle to find approximations for $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.



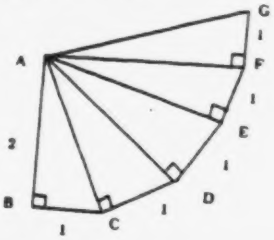
Note: This activity provides some good review of measurement techniques with both the protractor and the ruler. It is important to be accurate in order to get a good approximation.

Activity: 1.2 The Pythagorean Relation can be used to approximate $\sqrt{2}$ on a number line:



Note: Students could measure this to discover a numerical approximation of $\sqrt{2}$.

Problem: What is the length of segment AG?



Answer: 3 units.

Problem: Mental Calculation: How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

Answer: 6.

Problem: Express in simplest form:

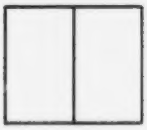
$$3 \cdot (8)^{2/3}\left(\frac{27}{8}\right)^{-1/3}$$

Answer: 8.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

Problem: Metric paper sizes have the property that a sheet of paper cut in half gives two smaller sheets of the same shape as the original sheet. Find the ratio of the sides of the sheets.

Answer: $l/w = \sqrt{2}/1$



Have the students verify this fact with various sizes of metric paper. For more information, see "Metric Paper", in Activities for Junior High School and Middle School Mathematics.

TECHNOLOGY INTEGRATION

Activity: Have students use their calculators to examine the relationships between the base and the exponent - for example, for what values of the exponent can you find the root of a negative number?

Problem: Have students use their calculators to solve this:

Which is larger, $\sqrt{10} + \sqrt{17}$ or $\sqrt{53}$?

Answer $\sqrt{10} + \sqrt{17}$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1987.

For an example of how spreadsheets can be used to help develop students' understanding of rational exponents, see "Integrating Spreadsheets into the Mathematics Classroom" pp. 619-620, Mathematics Teacher, November 1988.

ELECTIVE SUGGESTIONS

Problem: A glazier wished to bring a huge mirror which measured 2.5 m by 2.25 m into a house through a doorway which measured 2.1 m by 0.9 m. Could he get the mirror into the house in one piece?

Answer: Yes

Extension: Include the 4th and 5th roots, but only for those numbers who have easily identifiable 4th and 5th roots.

Problem: If the intensity of sunlight is reduced by ½ at a water depth of 1m, what is the intensity at a depth of 3.5m?

Answer: $(\frac{1}{2})^{3.5}$

Reprinted from Applications Elementary Algebra and Geometry, Applications in School Mathematics, National Council of Teachers of Mathematics, 1979.

RESOURCE CORRELATION

MMW3:	pp.28–30, 33–36
AM11:	pp. 44–54
MM3:	pp. 56–62
MPMM3:	pp. 28–30, 33–36

CONCEPT 2

Students will be expected to demonstrate an understanding of how interest accumulates if it is allowed to compound.

Students will be expected to:

2.1 calculate compound interest by using an iterative process

2.2 calculate compound interest using the formula $A = P(1 + i)^n$, where A is the amount (principal and interest), i is the interest rate per conversion period and n is the number of periods

  2.2.1 use a calculator and a spread sheet in the calculation of compound interest

  2.2.2 solve problems involving the calculation of compound interest

COMMENTS

It is meant that students will calculate compound interest by doing numerous simple interest calculations. A review of simple interest may be required. Students have discussed simple interest in junior high but it has not been reviewed since then. The intent is for students to begin to see the pattern in calculating compound interest

An introduction or review of terms associated with compound interest may be required. Terms such as principal, interest, rate, time (annual, monthly, bimonthly).

Introduction of the formula should only be done following the iterative process of calculating compound interest.

You may wish to check the present interest rates at your local financial institutions or by consulting the newspaper.

The study of compound interest should not include the present value of money or annuities.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students compute the amount of interest that a certain amount of money would earn over a set period of time if it was compounded annually; compounded semi–annually; compound daily. Prior to any calculations, have students predict which situation would earn the most interest. Students should perform the calculations and make observations regarding the amount of interest each is earning in specified times...e.g. which situation earned the most interest in a one month period? in a 2 week period? in a six month period? in a one year period? Example:

Is there much of a difference in the value of $1000 invested at a rate of 10%/a if the interest is compounded annually, monthly, weekly, daily, or even hourly?

This example could be extended to examine the differences between a certain amount of money being invested in a simple interest account for one year and then for ten years and compare that with the same amount of money invested in a compound interest account.

Problem: Cathy invests $500 at 8.5% compounded monthly. How long will it take for her investment to double?

Answer: One solution: write the expression,

$$500(1 + 0.00708333\ldots)^n = 1000$$

so $(1 + 0.00708333\ldots)^n = 2$ and then guess and test the value of n, the number of months required for Cathy's investment to double. An effective way of keeping track of the guesses is to set up a table. An effective way of guessing would be to create the table using a spreadsheet. (n = 99 months or 8 years and 3 months)

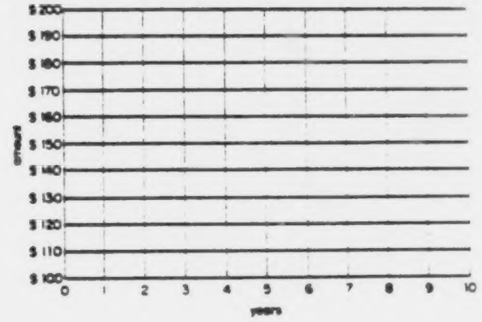
COMPOUNDING

1 Determine what happens to $100 compounded semi-annually at 10% over 4 years. Round down to the nearest cent.

Principal	Semiannual Rate	Interest	Amount		
$100.00	½(10%)	$5.00	$105.00		$1.05^1 \times 100$
$105.00	0.05	$5.25	$110.25	end of year 1	$1.05^2 \times 100$
$110.25	0.05	$5.51(25)	$115.76(25)		$1.05^3 \times 100$
	0.05			end of year 2	$1.05^4 \times 100$
	0.05				
	0.05			end of year 3	
	0.05				
	0.05			end of year 4	

Describe two methods by which you can determine the amount of money available at the end of 5 years.

2. Using o's, graph what happens to $100 invested at 10% simple interest over a period of 10 years. Using x's, graph what happens to $100 compounded semiannually at 10% interest per year over a period of 10 years. Use your answers to exercise 1 above. Discuss the results



Reprinted from "Compound Interest a la Simple Calculator," in Activities from the Mathematics Teacher, copyright 1981 by the National Council of Teachers of Mathematics..

See "Get Away from the Table! Make Interest More Interesting," Mathematics Teacher, December 1986, for a discussion, with examples, of using an iterative approach with a calculator to determine the interest rate in problems of compound interest, annuity amounts, present value, and so on.

TECHNOLOGY INTEGRATION

Students should use their calculators to perform the calculations.

A spreadsheet assists with the repeat calculations when calculating compounding interest by doing several simple interest calculations.

See also AM11: pp. 42-43, 472-474.

Spreadsheets are suggested as a way to calculate compound interest. Appleworks and Microsoft Works are Alberta Education Basic Resources and can be purchased from L.R.D.C.

ELECTIVE SUGGESTIONS

Problem: At 1.75% monthly interest rate, how much must be paid for a $300 chair if it is charged, and the purchaser waits m months to pay? Write the correct expression.

Answer: $300(1.0175)^{m}$

Problem: Interest on the security deposit that tenants must give the landlord is compounded annually. The schedule of interest rates in the past few years is as shown:

Time Period	Minimum Annual Interest Rate
Before December 31, 1981	6%
January 1, 1982 to December 31, 1983	12%
January 1, 1984 to February 28, 1987	8%
After March 1, 1987	6%

Guy, who has been in his apartment since January 1, 1982, is leaving at the end of December, 1989. When he first moved in, he paid a damage deposit of $300.00 how much will he receive in interest when he leaves?

Answer:

a) January 1, 1982 – December 31, 1983: 2 years; $i = 0.12$
$A = P(1 + i)^{n} = 300(1.12)^{2} = \$376.32$

b) January 1, 1984 – February 28, 1987: $3^{2}/_{12}$ years; $i = 0.08$
$A = P(1 + i)^{n} = 376.32(1.08)^{3.167} = \$480.18$

c) March 1, 1987 – December 31, 1989: $2^{10}/_{12}$ years; $i = 0.06$
$A = P(1 + i)^{n} = 480.18(1.06)^{2.833} = \$566.37$

Hence the amount of interest earned is
$\$566.37 - \$300.00 = \$266.37$

Note: Now compare this with the amount Guy would have earned if the interest was calculated as a simple interest problem where $r = 6\%$ and $t = 8$ years.
$[I = PRT = (300)(0.06)(8) = \$144.00]$

Extension: Examine the Present Value of a lump sum.

RESOURCE CORRELATION

MMW3:	pp. 214–221, 226–234, 457–458; 235–240 (extension)
AM11:	pp.42–43, 314–321, 342–355, 502–503; 472–474 356–359 (extension)
MM3:	pp. 66–74
MPMM3:	pp. 214–221, 226–234, 457–458; 235–240 (extension)

ALGEBRA
PROGRAM EMPHASIS – 10%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that equations and inequalities express mathematical relationships and can be used to model physical relationships.	Students will be expected to solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures.	Students will be expected to represent problems with equations and solve problems through the solution of the equations.
	Students will be expected to solve formulas for given subjects.	
	Students will be expected to solve, graph and verify the solution of first degree inequalities in one variable.	Students will be expected to represent problems with inequalities and solve problems through the solution of the inequalities.
Students will be expected to demonstrate an understanding that polynomials can be written as a product of their factors.	Students will be expected to factor (over the integers) trinomials of the form $ax^2 + bx + c$, $a, b, c \in I$.	
	Students will be expected to differentiate between quadratic expressions and quadratic equations.	
	Students will be expected to solve and verify the solution of quadratic equations that can be factored.	Students will be expected to use a graphing calculator or graphic computer software to recognize the relationship between the solutions to a quadratic equation and its corresponding graph.
	Students will be expected to solve and verify the solution of quadratic equations that can be reduced easily to $x^2 = a$.	Students will be expected to use a graphing calculator or graphic computer software to recognize the relationship between the solutions to a quadratic equation and its corresponding graph.

CONCEPT 1

Students will be expected to demonstrate an understanding that equations and inequalities express mathematical relationships and can be used to model physical relationships.

Students will be expected to:

1.1 solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures
    1.1.1 represent problems with equations and solve problems through the solution of the equations.
1.2 solve formulas for given subjects .

COMMENTS

The focus of this unit is on solving equations and inequations - firstly examining linear equations and inequalities and then extending the study to quadratic equations. It is also the intent of this unit to examine the "pictorial" representation of solving equations. For instance, what does it mean to solve a quadratic equation? Student should have the opportunity to explore the graph of a quadratic function in order to determine the information found when solving a quadratic equation. In solving linear equations, you may wish to have students graph their linear equations (in the general form) to identify when they are solving for a particular variable they are actually finding a point on the line.

Students have spent a great deal of time solving linear equations that involve all forms of rationals. Equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, $ax + bx = c$, $x/a = b/c$, $ax + b = cx$, $a(x + b) = c$, and $ax + b = cx + d$ have been studied in junior high. Equations of the form $a/x = b$ may not have been solved.

It is important to stress what an equation is: it is a statement of the equality that may be true, may be false or may depend on the values of the variable. An example of an equation that is true is $3+6=9$. An example of an equation that is false is $6+9=17$ and an example of an equation that depends on the values of the variable is $2+x=18$. This then helps to reinforce the concept of variable. Although x represents a number, it could be any number (so it varies) and in the process of solving the equation, a particular value for x is determined.

Students should be given ample opportunity to translate English into algebraic expressions and should have the opportunity to solve problems for which they can write equations. Solving problems by type: e.g., coin, mixture, age, distance problems should be avoided. Students should not experience these as "distinct" types of problems and that there is only one way to solve the problem.

Note: For a discussion of using diagrams to pictorially represent the solution to algebraic problems, see "Developing Algebraic Representation using Diagrams, "in The Ideas of Algebra, K-12, 1988 National Council of Teachers' of Mathematics Yearbook.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: To encourage flexible thinking in problem solving, have students solve equations for a specified algebraic expression rather than just for the variable. For example, solve $3(x + 4) - 8 = 13$ for $x + 4$ and solve $x + 7 = 10$ for $5x$, etc. This allows students to practice their solving equation skills, yet takes it one step farther to a more abstract form of the problem. Students may solve these problems by first solving for the variable and then substituting into the given expression. They can be led to solving for the expression by the use of formula manipulation, for example, solve $P = 2(1 + w)$ for $1 + w$.

Problem: Solve for: $(4x-7)/3$ if $6(4x-7) = 18$

Many students will want to solve for x, and then substitute in $(4x-7)/3$.

How many considered the following:

$$6(4x-7) = 18$$

$$6(4x-7)/18 = 18/18$$

$$(4x-7)/3 = 1$$

Provide other similar opportunities for the students.

Adapted with permission from "Flexibility and Algebraic Problem Solving," Mathematics Teacher, April 1987, copyright by the National Council of Teachers of Mathematics.

Problem: Find two numbers between $^2/_3$ and $1\ ^7/_{15}$ such that when the four numbers are arranged in order, the difference between any tow consecutive numbers will be the same.

Answer: $^{14}/_{15}$ and $1\ ^1/_{15}$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1988.

Activity: Consider the following class activity as a means of developing students' word number skill.

Create a deck of index cards, at least one card for each student in the class. On the front of each card write a number . On the back of each card, write instructions that involve computations with the present number that lead to a number on another card in the deck.

For example, three consecutive cards might be:

7 - who has twice this number less three?
11 - who has this number squared less 100?
21 - who has the square root of the sum of this number and 15?

Distribute all the cards. Designate a student to read his or her number and the instruction on the back. A second student, with the correct response to the instruction, should follow. The final card should contain an instruction that leads back to the first number read, this completing the chain.

In addition to encouraging the development of skills in translating from words to mathematical expressions, this activity also encourages mental and calculator computation.

To extend this activity, have students create another set of cards.

Adapted with permission from "I Have a Number . . .," pp. 550-551, Mathematics Teacher, October 1988.

Problem: The two equations below both have 2 as a solution:

$$(7p-2)/4 = 3 \qquad 3(17n-30) = 12$$

Create two equations that have the same form as these but have 9 as a solution. This emphasizes the strategy of working backwards when solving problems.

Reprinted with permission from "Activities: Teaching Problem-Solving Skills," in Activities for Implementing Curricular Themes from Agenda from Action.

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, pp.65, 66 #2, 19.

Problem Solving Challenge for Mathematics, pp.49, 50, #5, 20.

TECHNOLOGY INTEGRATION

For an example of using a spreadsheet approach to solving equations, see "Mathematical Modelling using Spreadsheets," delta–K, June 1989.

Software:
Equations II (Mindscape) provides structures remedial opportunities for solving equations of the form $Ax + B = Cx + D$.

Equations III (Mindscape) provides structured practice in solving equations of the form $Ax + B(Cx + D) = Ex + F$.

MMW3: pp.17–18.

See "Mathematical Modeling Using Spreadsheets," in delta–K June 1989, for a discussion of using spreadsheets to solve equations and to increase students' understanding of what "solving" an equation means.

Have students check their solution to equations using a calculator.

ELECTIVE SUGGESTIONS

Activity: It is a fact that the difference between two consecutive perfect squares is equal to the sum of their square roots. For example:

$$\begin{aligned} 121-100 &= \sqrt{121} + \sqrt{100} \\ &= 11 + 10 \\ &= 21 \end{aligned}$$

Have students investigate the patterns of difference between consecutive perfect squares and the sum of their square roots. Once the pattern indicated above has been established, have students investigate why this should be so generalizing. How can we represent the general case of two consecutive perfect squares? If n is any number, then n^2 and $(n+1)^2$ is one way.

Problem: Let a, b, c, and d be non–zero integers between −10 and 10. What are the largest and smallest solutions possible for $ax+b=cx+d$? This activity not only involves students in solving problems, but it will reinforce estimation and assist students in seeing patterns that may form in reaching this solution.

Answer: largest solution: $x=20$; smallest solution: $x=-20$.

Reprinted with permission from: "Reasons for Estimating", in Estimation and Mental Computation, copyright 1986 by the National Council of Teachers of Mathematics.

Activity: Students should understand that equations the terms of which are not identical may still be equivalent, that is, have the same solution. Shown below are 20 equations, some of which are equivalent. Have students use loops to make separate groups of equivalent equations so that all equations in the same group are equivalent to one another. (The figure below shows the correct grouping)

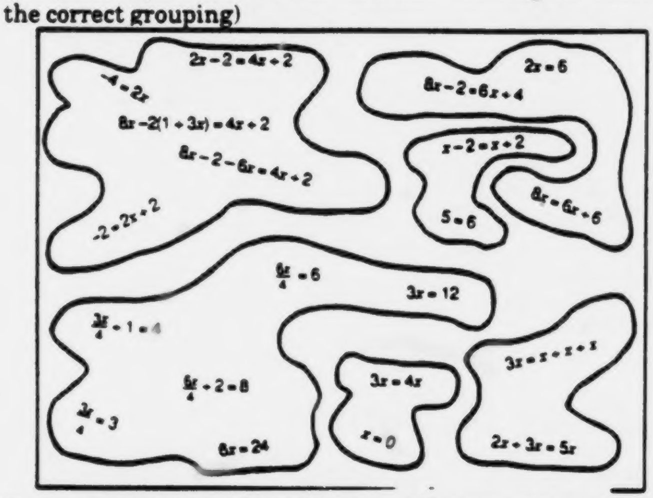


Figure represented from "An Integration of Equation-solving Methods into a Developmental Learning Sequence," The Ideas of Algebra, K–12, 1988 Yearbook of the National Council of Teachers of Mathematics.

See this article for a general discussion of equation solving methods.

RESOURCE CORRELATION

MMW3:	pp. 12–14, 19, 70–73, 76–83; 37–39 (extension), 44 (extension).
AM11	pp. 98–100, 102–106, 112–113, 156–157
MM3:	pp. 150–161
MPMM3:	pp. 12-14, 19, 70–73, 76–83; 37–39 (extension), 44 (extension)

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that equations and inequalities express mathematical relationships and can be used to model physical relationships.

Students will be expected to:

1.3 solve, graph and verify the solution of any first degree inequalities in one variable
   1.3.1 represent problems with inequalities and solve problems through the solution of the inequalities

COMMENTS

Students have some previous work with solving and graphing inequalities of the form $x+a \geq b$ and $cx \leq d$ (where c is positive and the direction of the inequalities vary). Some time has also been spent on verifying the solution to inequalities. Students have not been expected to solve an inequality by multiplying or dividing by a negative number.

"Inequality" problems can be "equality" problems that are adjusted to become inequality problems.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students write several true inequalities in involving pairs of rational numbers. To both sides of these inequalities apply the same operations. Check the results for truthfulness.

	+7	-9	×5	x(-6)	+2	+(-2)
6<10	13<17 True	-3<1 True	30<50 True	-30<-60 False	3<5 True	-3<-5 False
-3<6						
0<12						
-10>-20						
6>-3						

TECHNOLOGY INTEGRATION

Software:
Algebra Drill and Practice II (Conduit), options "Linear Inequalities" and "More Linear Inequalities" provide practice in solving inequalities.

ELECTIVE SUGGESTIONS

Problem: A telephone company charges $7 a month for each phone on a party line. The company has determined that the cost of servicing n phones on a party line is 2^n dollars per month. What is the maximum number of phones that can be installed on a party line before the company loses money?

Answer: $n = 5$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

Some solution strategies:

(a) Set up a table and systematically guess-and-check to determine the maximum n for which $7n \geq 2^n$.
(b) Use a computer spreadsheet program in a manner similar to (a).
(c) Use a computer to graph the functions $y = 2^n$ and $y = 7n$.

RESOURCE CORRELATION

MMW3:	pp.
AM11:	pp.
MM3	pp. 165–169
MPMM3:	pp.

CONCEPT 2

Students will be expected to demonstrate an understanding that polynomials can be written as a product of their factors.

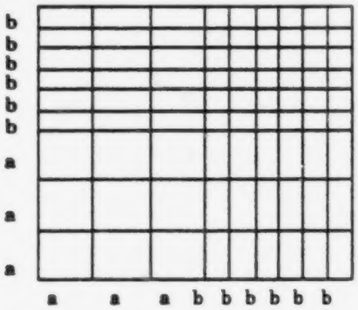
Students will be expected to:

2.1 factor (over the integers) trinomials of the form $ax^2 + bx + c$, a, b, c I

PROCESS/PROBLEM-SOLVING CONTEXT

Students were introduced to factoring trinomials of the form $ax^2 + bx + c$ in grade 10. Concrete and pictorial approaches to factoring using algebra tiles and a "binomial grid" are described in the Mathematics 10/13/14 interim Teacher's Resource Manual.

The binomial grid appears as:



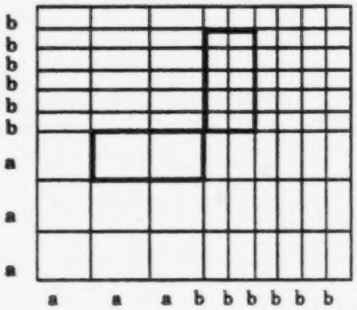
To represent negative elements, mark the areas affected in some way. A doubly marked area is interpreted as positive. To factor using the grid, one must identify a rectangular region to represent the given trinomial. The lengths of the sides of the rectangle represent the binomial factors.

Example: Factor $2x^2 - 9x + 10$

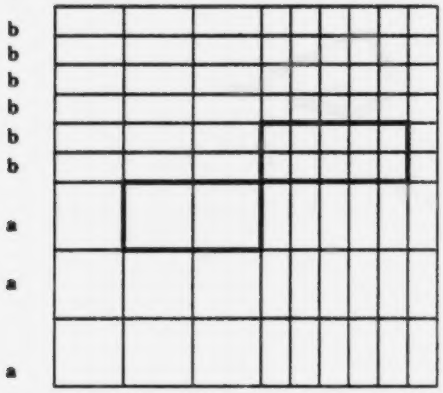
First determine the pairs of factors of the coefficient of the x^2 terms, and the constant term.

2: {(2,1)}
10: {(5,2), (10,1)}

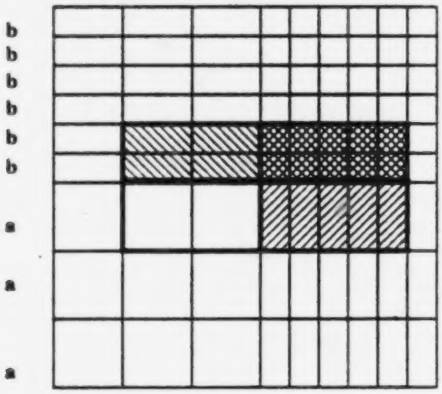
Start by using the (2,1) and (5,2) pairs to attempt to form the rectangle. (5,2) appears to offer more promise than (10,1). One possibility is:



A rectangle representing the given trinomial cannot be formed from this structure. The second possibility is:



The necessary rectangle can be formed:



Thus, $2x^2 - 9x + 10 = (2x-5)(x-2)$.

Adapted with permission from "The Binomial Grid," *Mathematics Teacher*, May 1986, copyright by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Software:
Solving Quadratic Equations and Factoring Algebraic Expressions (both through Mindscape) provide practice in factoring trinomial.

ELECTIVE SUGGESTIONS

Activity: Alternative Approaches to Factoring Trinomials of the Form $ax^2 + bx + c$..

a). consider the theorem which states:

Given a trinomial $ax^2 + bx + c$ and integers m and n such that $m + n = b$ and $mn = ac$, then

$$ax^2 + bx + c = \frac{(ax+m)(ax+n)}{a}$$

Encourage students to verify this statement with examples. For example, factor the expression $2x^2-7x-15$. In this case, $b = -7$ and $ac = -2 \times -15 = -30$. The procedure is to look for two numbers, the sum and products of which are respectively 7 and −30. These two numbers (m and n) are +3 and −10.

The theorem tells us that we can now directly write down the factored expression as:

$$2x^2 - 7x - 15 = \frac{(2x+3)(2x-10)}{2}$$

In this case, one of the binomials contains a common factor, 2. Simplifying, then, we have:

$$2x^2 - 7x - 15 = \frac{(2x+3)(2x-10)}{2} = (2x+3)(x-5)$$

As an activity, encourage the students to prove this theorem.

b) A second, related, theorem states:

Given a trinomial $ax^2 + bx + c$ and integers m and n such that $m + n = b$ and $mn = ac$, then

$$ax^2 + bx + c = \frac{(mx+c)(nx+c)}{c}$$

Have the students verify this theorem with examples. Using the previous example, $2x^2 - 7x - 15$, where $m = +3$ and $n = 10$. Substituting according to the theorem produces:

$$2x^2 - 7x - 15 = \frac{(3x-15)(-10x-15)}{-15}$$

$$= \frac{(3)(x-5)(-5)(2x+3)}{-15}$$

$$= (x-5)(2x+3)$$

As an activity, encourage the students to prove this theorem.

(a) and (b) adapted with permission from "Factoring Trinomials," Mathematics Teacher, January 1986, copyright by the National Council of Teachers of Mathematics.

c) Another approach makes use of a "box" or "call" technique. A multiplication example will help introduce the approach: Consider $(2x-1)(x+5)$

Arrange the box as shown and multiply cell-by-cell; then combine terms:

	x	$+5$
$2x$	① $2x^2$	② $10x$
-1	③ -1	④ -5

Therefore $(2x1)(x+5) = 2x^2 + 10x - x - 5 = 2x^2 + 9x - 5$.

Notice that:

1. the products of the diagonal elements are the same $(-10x^2)$;
2. cell 1 holds the x^2 term of the trinomial;
3. the sum of the cells 2 and 3 yields the term in x, and
4. cell 4 holds the constant.

Factoring:

Consider the previous example, $2x^2 - 7x - 15$. Set up the box and place the x^2 and constant term as shown:

① $2x^2$	
③	④ -15

Cells 2 and 3 contain terms in x, the sum of which is $-7x$ and the product of which is $-30x^2$. These terms are $+3x$ and $-10x$. Place them in cells 2 and 3, in no particular order.

① $2x^2$	② $+3x$
③ $-10x$	④ -15

Determine the common factors of each row and column and place them as shown:

	$2x$	$+3$
x	$2x^2$	$+3x$
-5	$-10x$	-15

Therefore $2x^2-7x-15=(2x+3)(x-5)$.

Note: Remove any common factor in the trinomial before preceding.

Adapted with permission from "Box Technique for Factoring," *Mathematics Teacher*, February 1987, copyright by the National Council of Teachers of Mathematics.

RESOURCE CORRELATION

MMW3:	pp. 58–65
AM11:	pp. 84–87
MM3:	pp. 125–138
MPMM3:	pp. 58–65

CONCEPT 2 (continued)

Students will be expected to **demonstrate an understanding that polynomials can be written as a product of their factors.**

Students will be expected to:

- 2.2 **differentiate between quadratic expressions and quadratic equations**
- 2.3 **solve and verify the solution of quadratic equations that can be factored**
  - 2.3.1 **use a graphing calculator or graphic computer software to recognize the relationship between the solutions to a quadratic equation and its corresponding graph**

COMMENTS

The intent of this expectation is to provide student with an understanding of why we factor quadratics. This also provides a visual representation to a quadratic and will provide students with some preliminary understanding of the quadratic function. Note though that students should use some form of technology to graph the quadratic function so that frustration with graphing does not become a block to the recognition of the connection between the graph and factoring the polynomial.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: What positive number is equal to its square added to its opposite?

Answer: 2.

Note: This problem could be solved numerically with guess-and-test. It can also be solved by setting up a simple, factorable quadratic. The value of the latter situation is that it represents the general case, showing that the only solution is 2. Does the condition that the number positive limit the number of solutions?

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1986.

Problem: A garden has an area of 240m^2. What are its dimensions if the length is 8m more than the width?

Answer: 20m by 12m.

Problem: Solve: $5x = {}^4/_x - 1$, Answer: $x = {}^4/_5, -1$

$$5x^2 = 4 - x$$

$$5x^2 + x - 4 = 0$$

$$(5x-4)(x+1) = 0$$

therefore $x = {}^4/_5;\ x = -1$

TECHNOLOGY INTEGRATION

Software:

Use a graphing program such as Equation Math (MECC), Green Globs and Graphing Equations (Sunburst) or Master Grapher (Addison – Wesley) to explore what happens to various factorable polynomials if one of the numerical coefficients is changed just a little. Does estimation have a place?

Solving Quadratic Equations (Mindscape) provides practice in solving quadratic equations by factoring.

Algebra Drill and Practice II (Conduit), "Roots of Quadratic Equations" option.

RESOURCE CORRELATION

MMW3:	pp. 74–75
AM11:	pp. 101, 107–109
MM3:	pp. 162–165
MPMM3:	pp. 74–75

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that equations and inequalities express mathematical relationships and can be used to model physical relationships.

Students will be expected to:

2.4 solve and verify the solution of simple quadratic equations which can be reduced easily to $x^2 = a$

- **2.4.1 use a graphing calculator or graphic computer software to recognize the relationship between the solutions to quadratic equation and its corresponding graph**

COMMENTS

Stress that quadratic equations will have 2 solutions and that radical equations may yield roots which will not solve the original equation. Limiting examples:

1. $x^2 = 20$
2. $(2x + 5)^2 = 25$

Zeros are the values of the variable which give the polynomial a value of 0. As a way of finding the zeros of a polynomial, introduce the following:

if $ab = 0$, then $a = 0$, or $b = 0$.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students use equivalences to demonstrate that squaring or taking the square root of both sides of an equation are legitimate operations. E.g.

$$5 = {}^{10}/_{2}$$

$$5^2 = ({}^{10}/_{2})^2$$

$$25 = {}^{100}/_{4}$$

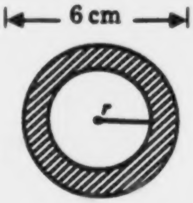
Problem: When I open my mathematics book, there are two pages that face me. If the product of the two pages is 462, what are the two page numbers?

Answer: page 21 and 22

Problem: The sum of twice a fraction and half that fraction, times that fraction equals that fraction. What is that fraction?

Answer: 0, or ${}^{2}/_{5}$

Problem: Find r such that the areas of the washer and the hold are equal.



Answer: $r = \frac{3\sqrt{2}}{2}$ cm

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, May 1988.

For more **Problems** see: Problem Solving in Mathematics: Focus for the Future, pp. 65, 66, #10, 13, (These problems involve rational expressions); p. 69, #25.

Problem Solving Challenge for Mathematics, p. 49, #1, 2 (involves rational expression).

TECHNOLOGY INTEGRATION

Algebra Drill and Practice II (Conduit), "Roots of Quadratic Equation" option.

Software:
Use a graphing program such as Equation Math (MECC), Green Globs and Graphing Equations (Sunburst) or Master Grapher (Addison – Wesley) to explore what happens to various factorable polynomials if one of the numerical coefficients is changed just a little. Does estimation have a place?

Solving Quadratic Equations (Mindscape) provides practice in solving quadratic equations by factoring.

ELECTIVE SUGGESTIONS

Problem: This activity will help students develop a sense of "reasonableness" with their algebra:

A cow and a rooster want to play on a teeter–totter. The rooster says its impossible, but the cow says that it will work and that he can prove it, since he's had a little algebra. The following argument is presented:

Let C = weight of cow
Let R = weight of rooster
then some W exists so that $C = R + W$
multiply both sides by $C - R$
$C(C-R) = (R+W)(C-R)$
so $C^2 - CR = RC - R^2 + WC - WR$
factor $C(C-R-W) = R(C-R-W)$
divide both sides by $(C-R-W)$ and then $C = R$
therefore the weight of the cow equals the weight of the rooster and the won't be a problem with the rooster and the cow playing on the teeter totter. Is the proof correct? Is the conclusion correct? Does the conclusion make sense? Where is the fault in the "proof"?

Problem: If x and y are consecutive odd integers, $x<y$. When $3x^2-2y = 129$, find the value of $x + y$.

Answer:

$y = x + 2$

$3x^2-2y = 129$

therefore $3x^2-2(x+2) = 129$

$3x^2-2x-133 = 0$

$(3x + 19)(x-7) = 0$

therefore $x = 7, y = 9$ and $x + y = 16$

Problem: Find the values of x which satisfy the following:

$$(x^2-2x+1)^{(x^2-4x+3)} = 1$$

Possible Solutions:

a) $x^2-4x+3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

(When $x = 3$, have 4^0

and when $x = 1$, have 0^0 so

$x = 1$ is not a solution)

b) $x^2-2x+1 = 1$

$x^2-2x = 0$

$x(x-2) = 0$

$x = 2$ or $x = 0$

(When $x = 0$, have 1^3

and when $x = 2$, have 1^{-1})

c) $x^2-2x+1 = -1$ and

x^2-4x+3 is even

but $x^2-2x+2=0$
does not factor

d) Other possibilities?

RESOURCE CORRELATION

MMW3:	pp. 74–75
AM11:	pp. 101, 107–109
MM3:	pp. 162–165
MPMM3:	pp. 74–75

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LINEAR RELATIONS
PROGRAM EMPHASIS – 10%

Concept	Skills	Problem Solving/ Technology
Students will be expected to be able to demonstrate an understanding that the equations and graphs of linear relations can be determined uniquely from given characteristics of the relation.	Students will be expected to write the equation and draw the graph of a linear given its slope and the y-intercept.	
	Students will be expected to write the equation and draw the graph of a linear relation given any two points.	
	Students will be expected to write the equation and draw the graph of a linear relation, given a point and a slope.	
	Students will be expected to write the equation and draw the graph of a linear relation given a point and the equation of a parallel or perpendicular line.	
	Students will be expected to write the equation and draw the graph of a linear relation given any combination of characteristics that uniquely define it.	Students will be expected to use calculators or computers to draw graphs of linear relations. Students will be expected to solve problems which involve writing the equations or drawing the graphs of linear relations. Students will be expected to interpret the equations of linear relations which represent physical phenomena.

CONCEPT 1

Students will be expected to demonstrate an understanding that the equations and graphs of linear relations can be determined uniquely from given characteristics of the relation.

Students will be expected to :

1.1 write the equation and draw the graph of a linear relation given its slope and the y–intercept
1.2 write the equation and draw the graph of a linear relation given any two points
1.3 write the equation and draw the graph of a linear relation given a point and a slope
1.4 write the equation and draw the graph of a linear relation given a point and the equation of a parallel or perpendicular line

COMMENTS

This unit is an extension of the study of linear relations in Mathematics 13. In Mathematics 13, students study the distance between two points, the midpoint of a line segment, graphing linear relations, the slope and the relationship of the slope to horizontal, vertical, perpendicular, and parallel lines. You may wish to review these concepts prior to beginning this study.

Note: There is no specific mention of what form of the linear equation that students should be able to write, i.e., general form or slope–intercept form. Which form you will require of your students is entirely up to you. However, from Mathematics 13 and a good extension of the Mathematics 23 unit on algebra, students should understand that they can transfer either form of the equation to the other form and that these forms are equivalent.

Note 1.2: "Any two points" includes the x- and y-intercepts.

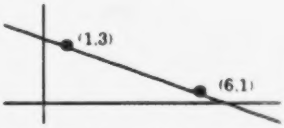
Students should work with graphing the lines given conditions prior to writing the equations. From here, students could write the equation of line by reading the required information from the graph. Finally, students could be given the conditions, graph the line and write the equation. These should help to reinforce the graphing skills previously learned.

Students should be able to sketch the graph of every condition not given in graphical form.

Limiting Examples:

Determine the equations of the following lines:

a)



b) with a slope of -2 and a y–intercept of -6
c) through $(5,-6)$ with a slope of $^1/_3$

d) through (−4, 2) and (5, −8)
e) through (−3, −2) parallel (or perpendicular) to $2x + 3y = 9$

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students determine the necessary and sufficient conditions to define a line. When they have suggested the need for a point and a slope, determine the point and slope which have been given as conditions in the problem. Introduce the general point (x, y) as an auxiliary element which represents every point on the line. Can an equation be written which describes the conditions between the given point, the general point and the slope? Can the equation be simplified and does it represent the initial conditions?

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Software:
Master Grapher Addison-Wesley), "Function Grapher" option. Equations must first be written as a function of x.

Algebra Drill and Practice I (Conduit), "Slopes and Equations of lines" option, provides practice in writing the quotation of a line given the slope and a point, or two points.

Computer Graphing Experiments I (Addison-Wesley). Using the "Linear Equations" option "Graphs of the form $y = mx + b$" and the accompanying worksheets, students may write and graph equations given various conditions.

Equation Math (MECC) -"Linear" option. Equations written in $y = mx + b$ form may be input.

Green Globs and Graphing Equations (Sunburst) - "Linear and Quadratic Graphs" option. In linear mode, this courseware provides students with the opportunity to develop their ability to interpret and write the equation for given straight line graphs.

See "Mathematics Modeling Using Spreadsheets", in delta-K June 1989, for a discussion of how a spreadsheet (with graphing capabilities) may be used to develop an understanding of writing the equation for, and graphing, a linear relation.

RESOURCE CORRELATION

MMW3:	pp. 117–119, 121–126
AM11	pp. 131–139
MM3:	pp. 191–194
MPMM3:	pp. 117–119, 121–126

CONCEPT 1 (continued)

Students will be expected to **demonstrate an understanding that the equations and graphs of linear relations can be determined uniquely from given characteristics of the relation.**

Students will be expected to:

1.5 **write the equation and draw the graph of a linear relation given any combination of characteristics which uniquely define it**
  - 1.5.1 **use calculators or computers to draw graphs of linear relations**
  - 1.5.2 **solve problems which involve writing the equations or drawing the graphs of linear relations**
  - 15.3. interpret the equations of linear relations which represent physical phenomena

COMMENTS

This particular expectation is an extension of the work in Mathematics 13 that describes graphing graphs that describe "real-world phenomena", for example drawing the graph of the temperature of water over time when an ice cube is dropped into a glass of water. In this expectation, it is expected that students will examine some direct, inverse, and partial variation problems that can be graphed and solved, using the graphs. From these graphs, then the "practical significance of the parameters" can be examined. For instance, if you were to graph a graph for the total distance travelled in a specific number of hours when the speed of the vehicle is 100 km/h and the vehicle started 45 km away from a base ($d=100t+45$): what does the 100 represent on the line (the slope) and what does the 45 represent (the y intercept). A second example: if the equation relating the distance travelled (d kilometres) after a certain time (t hours) for a particular car is given as $d=80t - 160$: what significance is the "80" and the "-160"?
(Thanks to H. Hohol, McNally C.H.S.)

TECHNOLOGY INTEGRATION

Computers and graphing calculators can be used to graph the linear equations.

Software:
Master Grapher (Addison-Wesley), "Function Grapher" option. Equations must first be written as a function of x.

Algebra Drill and Practice I (Conduit), "Slopes and Equations of lines" option, provides practice in writing the equation of a line given the slope and a point, or two points.

Computer Graphing Experiments I (Addison-Wesley). Using the "Linear Equations" option "Graphs of the form $y=mx + b$" and the accompanying worksheets, students may write and graph equations given various conditions.

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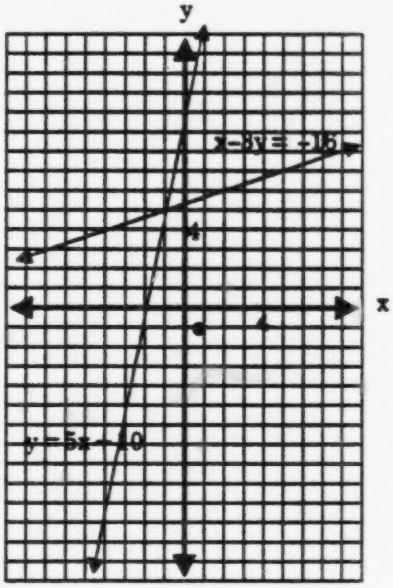
Equation Math (MECC) - "Linear" option. Equations written in y = mx + b form amy be input.

Green Globs and Graphing Equations (Sunburst) - "Linear and Quadratic Graphs" option. In linear mode, this courseware provides students with the opportunity to develop their ability to interpret and write the equation for given straight line graphs.

ELECTIVE SUGGESTIONS

Problem: A Time Air plane is flying towards the Calgary International Airport on a course defined by the equation $y=5x+10$. A Canadian Airlines jet is also flying towards the Calgary airport. Its path is described by the equation $x-3y=-16$.



1. At what point is the Calgary International Airport located?
2. Verify that each plane is on a path that will bring it to its destination.
3. Will the planes collide with one another? Why or why not?

(Thanks to L. Hovdebo, Sir Winston Churchill H.S.)

RESOURCE CORRELATION

MMW3:	pp. 102–105 (except non-linear relations), 123–124, 380–387
AM11:	pp. 140–144 (except inverse variation)
MM3:	pp. 179
MPMM3:	pp. 102–105 (except non-linear relations), pp. 223–124, 380–387

SYSTEMS OF EQUATIONS
PROGRAM EMPHASIS – 13%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two equations in two unknowns.	Students will be expected to solve systems of linear equations in two unknowns graphically.	Students will be expected to use a graphing calculator or graphic computer software to graph two linear equations.
		Students will be expected to solve problems involving the graphs of two linear equations.
	Students will be expected to solve systems of linear equations in two unknowns algebraically.	Students will be expected to solve problems that can be represented by a system of two linear equations in two unknowns.

CONCEPT 1

Students will be expected to demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two equations in two unknowns.

Students will be expected to:

1.1 solve systems of linear equations in two unknowns graphically
  1.1.1 use a graphing calculator or graphic computer software to graph two linear equations
  1.1.2 solve problems involving the graphs of two linear equations

COMMENTS

It is important to graph systems of equations to demonstrate to students that when solving a system that an intersection point will be the only point which satisfies (or solves) both equations. If the lines do not intersect, then there is no solution.

Reinforce for students that the coordinates of the points which make up a line are solutions for the equation of the line. Thus, the intersection point of two lines will be the only point which satisfies (solves) both equations. Impress upon students the importance of checking in both equations.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students draw the graphs of both equations in a given independent system. Discuss the importance of the point of intersection and then verify that it does satisfy both equations in the system.

Activity: Have students sketch the graphs of the equations in a system on the same coordinate plane. Have them add and subtract the systems and graph the equations of the results. Have them multiply one or both of the equations, add or subtract the changed equations and graph. Why do all of the equations pass through the same point? Are there any restrictions on what can be done to the equations in a system and still maintain the graph through the same point? How could an equivalent system be created which had only lines parallel to the x– and y–axes?

Problem: Find the area of the triangle determined by the x-axis, the line with equation $y = x$, and the line with equation $x + 3y = 12$.

Answer: The vertices of the triangle are (0,0), (3,3) and (12,0). The triangle has a base of 12 and an altitude of 3. The area is 18 square units.

TECHNOLOGY INTEGRATION

For an example of how spreadsheets can be used to help develop students' understanding of systems of linear equations, see "Integrating Spreadsheets into the Mathematics Classroom", pp. 617–619, Mathematics Teacher, November 1988.

Software:
Master Grapher (Addison-Wesley), "Function Grapher" option.

Algebra Drill and Practice II (Conduit) - "Linear Systems in Two Variables" option.

Computer Graphing Experiments I (Addison-Wesley) - "Linear Equations - Simultaneous Equations" option.

Equation Math (MECC) -"Exploring Equations" option.

Green Globs and Graphing Equations (Sunburst) - "Equation Plotter" option.

RESOURCE CORRELATION

MMW3:	pp. 132–135
AM11:	pp. 148–149
MM3:	pp. 198–200
MPMM3:	pp. 132–135

CONCEPT 2

Students will be expected to demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two equations in two unknowns.

Students will be expected to:

1.2 solve systems of linear equations in two unknowns algebraically

  1.2.1 solve problems which can be represented by a system of two linear equations in two unknowns

COMMENTS

Solving systems graphically may provide an inaccurate estimate of the solution, hence, it is convenient to use an algebraic method in order to obtain an exact solution.

Addition/Subtraction:
It is important that students understand why algebraic methods work. Remind them of the processes involved in solving equations, i.e., that performing the same operation on both sides of an equation creates an equivalent equation. Use this fact to demonstrate that the sum or difference of a system is satisfied by the same point as the system. This examination provides a reason why the addition/subtraction algorithm works.

Provide students with a method for keeping track of the equations and operations on them. The following is a suggestion:

$3x + 4y = 29$ ①

$2x - 5y = -19$ ②

① x 5 $15x\ 20y = 145$

② x 4 $8x - 20y = -76$

Substitution:
Remind students that the reason for solving systems is to find the coordinates of the point which satisfied both equations. At that point, the values of x and y for both equations are the same. For this reason, it is legitimate to solve for one of the variables and then to substitute the expression for that variable into the other equation.

Stress that the reason for learning different methods of solution is for efficiency. Little is to be gained by forcing students to use one method instead of another unless they can see an advantage.

Provide students with lots of practice in writing equations to represent situations. Do not attempt to categorize problems by type.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: Aardvaark Auto Rentals changes $20/day plus $0.15/km to rent a new Thunderer. U–Drive Car Rentals charges $15/day plus $0.20/km. What must be considered before a decision is made as to which agency to use? What are the conditions under which each of the agencies would be chosen?

Activity: For an engaging idea that will assist students in developing their skills in graphically and algebraically solving systems of linear equations, and which can be extended to involve students in problem solving, see "Secret Codes and Systems of Equations", Mathematics Teacher, April 1986.

For more **Problems** see: Problem Solving in Mathematics: Focus for The Future, Alberta Education, pp. 65, 66 #15, 22; p. 67, #8.

Activity: As an activity to extend the concept of adding two equations, an examination of how the systems are added on the coordinate plane would be appropriate.

TECHNOLOGY INTEGRATION

For an example of how spreadsheets can be used to help develop students' understanding of systems of linear equations, see "Integrating Spreadsheets into the Mathematics Classroom", pp. 617–619, Mathematics Teacher, November 1988.

Software:
Master Grapher (Addison–Wesley), "Function Grapher" option.

Algebra Drill and Practice II (Conduit) – "Linear Systems in Two Variables" option.

Computer Graphing Experiments I (Addison–Wesley) – "Linear Equations – Simultaneous Equations" option.

Equation Math (MECC) –"Exploring Equations" option.

Green Globs and Graphing Equations (Sunburst) – "Equation Plotter" option.

ELECTIVE SUGGESTIONS

Problem: Suppose the government was to consider the following two unemployment compensation plans:

Plan A: Pay 50 percent of the average weekly salary with a maximum benefit of $160 a week.

Plan B: Pays a base amount of $75 plus 20 percent of the worker's average weekly salary with a maximum benefit of $160 a week.

Have students consider:

What questions come to mind in comparing the two programs?

Describe a mathematical model (of two equations in two unknowns) for comparing the two programs. Use it to answer some of the formulated questions.

Adapted with permission from "Wildlife, Unemployment and Insects: Mathematical Modeling in Elementary Algebra" in Applications in School Mathematics copyright 1979 by the National Council of Teachers of Mathematics.

RESOURCE CORRELATION

MMW3:	pp. 136–143
AM11:	pp. 150–155, 158–162
MM3:	pp. 201–218
MPMM3:	pp. 136–143

PROBABILITY
PROGRAM EMPHASIS – 11%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number from 0 to 1.	Students will be expected to use the language of probability.	
	Students will be expected to determine the sample space for simple events.	
	Students will be expected to determine the probability of simple events through experiments.	Students will be expected to carry out an investigation to determine the experimental probability of an event.
	Students will be expected to determine the theoretical probability of events which have easily countable sample spaces.	Students will be expected to compare theoretical probabilities to those for the same event generated empirically.
Students will be expected to demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.	Students will be expected to find the probability of two or more events occurring together by the application of the multiplication law for independent events, $P(A \text{ and } B) = P(A) \times P(B)$, and for dependent events, $P(A \text{ and } B) = P(A) \times P(B \mid A)$.	Students will be expected to solve problems which involve finding the probability of the occurrence of two or more events. Students will be expected to determine experimentally the probability of two or more events occurring together and compare it to the theoretical probability.

CONCEPT 1

Students will be expected to demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number from 0 to 1.

Students will be expected to:

1.1 use the language of probability
1.2 determine the sample space for simple events
1.3 determine the probability of simple events through experiments
    1.3.1 carry out an investigation to determine the experimental probability of an event
1.4 determine the theoretical probability of events which have easily countable sample spaces
    1.4.1 compare theoretical probabilities to those for the same event generated empirically

COMMENTS

Students have done some previous work with probability – in Grades 8 and 9. It is the intent that probability be introduced here and then how probability is used in simulating real world situations. Students should experience the situations first before working with the theoretical situations.

Note: See "Out of Thin Air", in Activities for Implementing Curricular Themes from the Agenda for Action for a set of activities, using thumbtacks, designed to develop students' understanding of experimental and theoretical probability, and theoretical probability, and their relationship.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity:

1) Thumbtacks: Have students speculate on the probability of a thumbtack landing point-up. Give each student a thumbtack and have them flip it ten times, keeping track of "point-up" and "side" occurrences. Record the relative frequency of the "point-up" event for each student , and construct accumulative relative frequency graph. Compare and discuss the speculative and experimental probabilities.

2) Cylinders: Have students each bring in three sturdy cylinders of various sizes (paper towel tubes, clean metal cans with the labels and ends removed and so on). Mark the ends "heads" and "tails". Have students determine the height to diameter rates, and use this value to classify the cylinders.

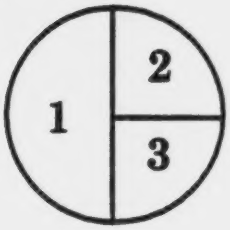
Speculate on the probability of each cylinder landing on its "head", "tail", or side when flipped. Flip each of the cylinders twenty times, recording the frequencies of the three landing orientation possibilities.

Were there any unfair heads/tails cylinders? Is there a pattern of landing emerging related to the ratio classification? What is the height to diameter ratio of a coin? What is the likelihood of it landing on its side?

Adapted with permission from "Experiential Statistics and Probability for Elementary Teachers", in Teaching Statistics and Probability copyright 1981 by the National Council of Teachers of Mathematics

Activity: Have the students lay a paper clip on Circle A holding one end of it at the centre of the circle with the tip of a pencil. Use the other hand to spin the paper clip 24 times. Record the results in Table 1 below. Divide the totals by 24 to compute the experimental probabilities. Then repeat the entire process using Circle B and Table 2.

A



B

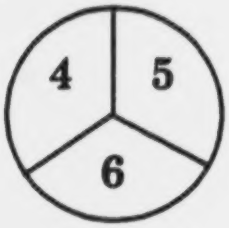


Table 1

Section	Tallies	Totals	Experimental Probabilities
1			
2			
3			
		24	1

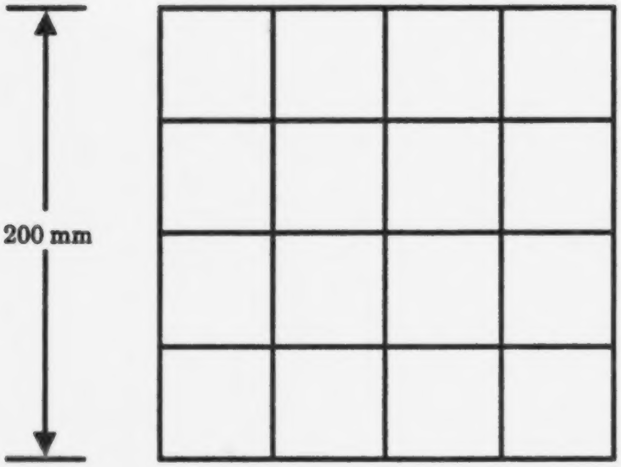
Table 2

Section	Tallies	Totals	Experimental Probabilities
4			
5			
6			
		24	1

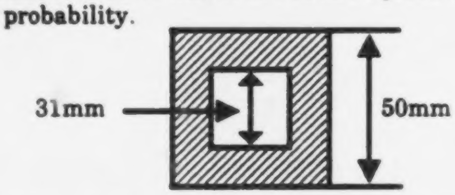
Have the students then determine the theoretical probability of obtaining a 1, 2, 3, 4, 5, or 6, and compare these results with experimental probabilities.

Create a grid "ruled" in 50mm squares, as shown. Randomly toss 5 pennies on the grid and count the number falling on a line. Repeat the experiment for a total of 20 times. Divide the total number of pennies falling on a line (hits) to the number tossed (100). Use this ratio as an estimation of the probability that a penny, randomly tossed on this grid, will fall on a line.

Total number of hits: ___
Total number tossed: ___
Probability of a hit: ___



Compare the results to the theoretical probability. One approach to determining this value is to consider the centre point of the 19mm penny and a single 50mm square. If the centre point falls in the shaded region, the penny lands on a line of the grid. Compare this shaded area to that of the entire square to find the corresponding probability.



Adapted from "Probability by Experimentation", Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy Series software "Exploring Probability" option.

Video:
Of Dice and Men (NFB).

RESOURCE CORRELATION

MMW3:	
AM11:	
MM3:	pp. 262–269
MPMM3:	
EP:	pp. 1–30

CONCEPT 2

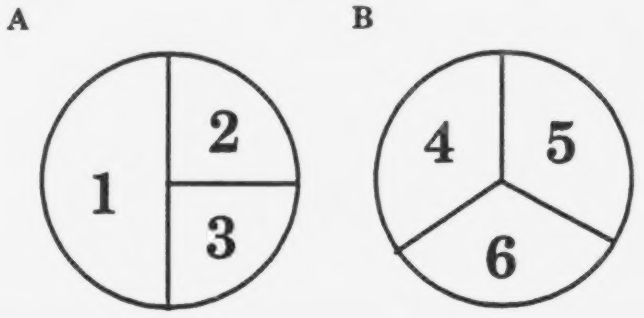
Students will be expected to **demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.**

Students will be expected to:

2.1 **find the probability of two or more events occurring together by the application of the multiplication law for independent events, P(A and B) = P(A) x P(B), and for dependent events, P(A and B) = P(A) x P(B|A)**
  - 2.1.1 **solve problems which involve finding the probability of the occurrence of two or more event**
  - 2.1.2 **determine experimentally the probability of two or more events occurring together and compare it to the theoretical probability**

PROCESS/PROBLEM-SOLVING CONTEXT

Activity:



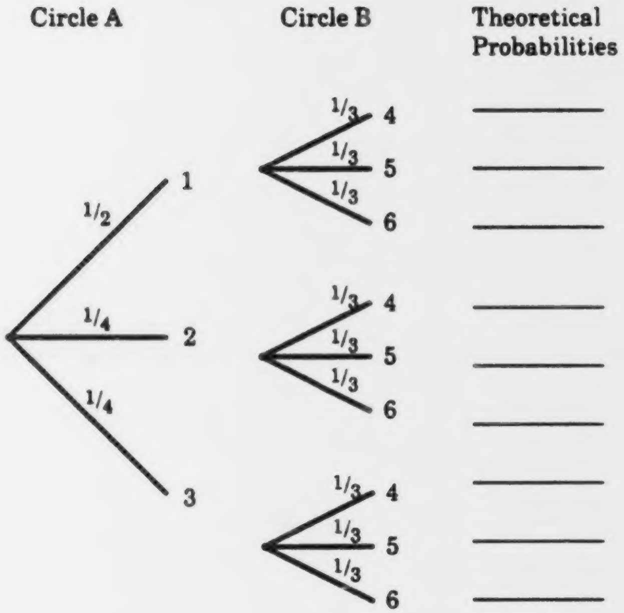
Using a paper clip and a pencil as a spinner as in the activity in Concept 1, page 168, alternate between circles A and B, making 24 pairs of spins. After each pair of spins on Circles A and B, add the two results to get a sum. Record these results in Table 3. Again, divide the totals by 24 to compute the experimental probabilities.

Table 3

Section	Tallies	Totals	Experimental Probabilities

Table 4

Sum	Theoretical Probabilities
5	
6	
7	
8	
9	
	1


Circle A
Circle B
Theoretical Probabilities
1/2
1/4
1/4
1
2
3
1/3
1/3
1/3
4
5
6
1/3
1/3
1/3
4
5
6
1/3
1/3
1/3
4
5
6

This tree diagram has nine branches representing the nine different number pairs possible on the two circles, A and B. Multiply the two fractions on each branch to find the theoretical probability for that number pair. Enter these probabilities in the spaces provided. To find the theoretical probability for each possible sum on the two circles, add the appropriate probabilities from above. Enter the results in table 4 above and compare these theoretical probabilities to the experimental probabilities listed in table 3.

Reprinted from "Experimental and Theoretical Probabilities," Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Problem: Each integer 1 through 9 is written on a separate slip of paper, and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer.

A) 0 B) 1 C) 8 D) 9

E) each digit is equally likely

Answer: A) 0

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

TECHNOLOGY INTEGRATION

Software:
Quantitative Literacy Series (American Statistical Association) "Exploring Probability" option.

ELECTIVE SUGGESTIONS

Extension: An extension is to find the probability of one or the other of two events, A and B by the application of the addition law, P(A or B) = P(A) + P(B) for mutually exclusive events and in general, (P(A or B) = P(A) + P(B) − P(A and B).

Problem: An ordinary deck of playing cards is shuffled. What is the probability that the first card drawn from the deck is either a black card or an ace?

One solution:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= {}^{26}/_{52} + {}^{4}/_{52} - {}^{2}/_{52} \\ &= {}^{28}/_{52} \\ &= {}^{7}/_{13} \end{aligned}$$

RESOURCE CORRELATION

MMW3:
AM11:
MM3: pp. 270–273
MPMM3:
EP: pp. 31–42

GEOMETRY
PROGRAM EMPHASIS – 13%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that within closed two-dimensional figures, there are relationships amongst the sides and angles.	Students will be expected to determine patterns which can be used to discover relationships pertaining to closed polygons.	Students will be expected to determine the sum of the interior angles of any polygon. Students will be expected to determine the relationship between the number of sides and diagonals in a polygon.
	Students will be expected to determine patterns to develop a formula for perimeter, which is the distance around a figure or an object.	Students will be expected to calculate the perimeter of any regular polygon. Students will be expected to solve problems involving the polygons. Students will be expected to determine that the ratio between the circumference and diameter of a circle is constant. Students will be expected to solve problems involving the circumference of circles.
	Students will be expected to determine patterns to develop a formula for area that is a measure of the number of non-overlapping square units which can be enclosed within a figure.	Students will be expected to calculate the area of any regular polygon. Students will be expected to calculate the areas of irregular regions that can be subdivided into regular polygons. Students will be expected to solve problems involving the area of polygons. Students will be expected to determine what figure encloses the largest area for a given perimeter. Students will be expected to calculate the area of a circle.

GEOMETRY (CONT'D)
PROGRAM EMPHASIS – 13%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that within three-dimensional figures, there are relationships amongst the sides, edges and vertices.	Students will be expected to determine patterns which can be used to discover relationships pertaining to three-dimensional figures.	Students will be expected to determine the relationship between the number of edges, vertices and faces in a simple polyhedron (Euler's formula $V - E + F = 2$). Students will be expected to determine the relationship between the number of faces and diagonals in a polyhedron.
	Students will be expected to determine patterns to develop a formula for the surface area of a rectangular solid, cone, sphere, cylinder, and pyramid.	Students will be expected to calculate the surface area of a rectangular solid, cone, sphere, cylinder, and pyramid. Students will be expected to calculate the surface area of irregular solids that can be subdivided. Students will be expected to solve problems involving surface area or regular and irregular solids.
	Students will be expected to determine patterns to develop a formula for the volume of regular solids (area of base times height).	Students will be expected to calculate the volume of regular solids (area of base times height). Students will be expected to calculate the volume of irregular solids that can be subdivided. Students will be expected to solve problems involving the volume of regular and irregular solids. Students will be expected to determine what figure encloses the greatest volume for a given surface area.

CONCEPT 1

Students will be expected to demonstrate an understanding that within closed two-dimensional figures, there are relationships amongst the sides and angles.

Students will be expected:

1.1 determine patterns which can be used to discover relationships pertaining to closed polygons
  1.1.1 determine the sum of the interior angles of any polygon
  1.1.2 determine the relationship between the number of sides and diagonals in a polygon

COMMENTS

It is the intent of this unit to extend students' understanding of the relationships that occur within two- and three-dimensional figures. Students study area, perimeter, volume and surface area of regular polygons and regular polyhedron in junior high. This unit is designed so that students understand that other relationships occur within these figures and that the formulas for each of the measures were not made up, but are actually developed by examining patterns. In addition, the material is extended to include irregular polygons and polyhedron.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Interior Angles of Polygons – students initial discussion of this should be with triangles. Students should measure, using the protractor the number of degrees in several "types" (e.g. scalene, right, obtuse, acute) of triangles. This will give the student practice in recognizing where measurements are taken when measuring the degrees of a polygon. The investigation should then be extended to quadrilaterals – with eventual reference made to the fact that a quadrilateral can be divided into two triangles. Further extension should be made to the remainder of the polygons, initially measuring the angles and then comparing it to the number of triangles that can be constructed within the polygon. Have students consider the two approaches shown. What difference if any, are there in these procedures? By the end of the investigation, students should have discovered "the rule" and be able to find the number of degrees in a polygon of n sides.

See "Polygons made to Order," Mathematics Teacher, January 1987 for a series of activities designed to help students understand the relationships among the angles of polygons, and draw regular polygons.

Problem: How many diagonals does a convex twelve-sided polygon have.

Answer: 54

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, February 1988.

Problem: Given a square, how can you demonstrate with a piece of construction paper that the sum of the angels equals 360° without using a protractor?

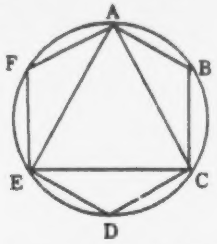
Answer: Fold the four vertice in to meet at the centre. The sides will meet with no overlap $4 \times 90° = 360°$

(What shape is formed when the vertices are folded in?

- Another square. What is the ratio of the length of side of the new square to the original? $1:\sqrt{2}$)

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, May 1987.

Problem: An equilateral triangle and a regular hexagon are both inscribed in the same circle, as shown. What is the ratio of the area of the hexagon to the area of the triangle?



Answer: 2:1

One approach: Use knowledge of the interior angles and area of regular polygons, and congruence postulates to demonstrate the ratio.

Activity: or more ideas see "Polygons Made to Order," Mathematics Teacher, January 1987, for a set of activities designed to develop students' ability to draw polygons, and to determine the measure of the central and base angles associated with each figure.

Activity 1.1.2: Have students complete a chart such as the following to examine the pattern for determining the relationship between the number of diagonals and the number of sides in a polygon.
(Thanks to L. Hovdebo, Sir Winston Churchill High School)

Polygon	Sketch	Number of Sides	Number of Diagonals
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Septagon			
Octagon			
Nonagon			
Decagon			

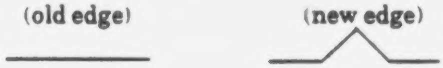
Determine a formula for calculating the number of diagonals for a given polygon. What do you think the number of diagonals will be for a dodecagon? Is there some way that you can "test" your answer?

TECHNOLOGY INTEGRATION

See "Investigating Shapes, Formulas, and Properties with Logo," in Activities for Implementing Curricular Themes from the 'Agenda for Action' for a set of activities in which students may explore characteristics of squares, rectangles, parallelograms and triangles using LOGO. The attraction of this activity for Math 23 lies principally in the LOGO experience.

ELECTIVE SUGGESTIONS

Activity: An example of fractal mathematics. Snowflakes may be created by forming an equilateral triangle on the middle third of every existing boundary segment. That is,,



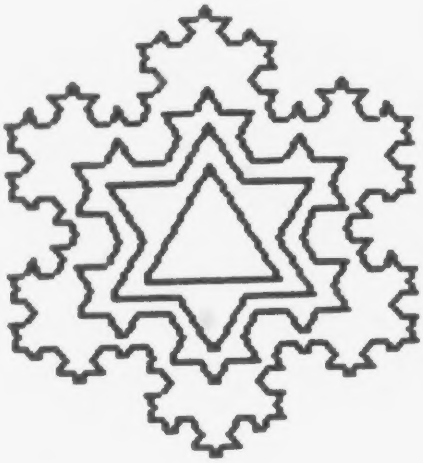
The following LOGO program will generate the first four stages of the snowflake to get the composite picture shown, run TRI.FRACTAL.T four times in succession without clearing the screen for these values of :x :y

40	0
60	1
80	2
120	3

```
TO TRI.FRACTAL.T :X :Y
   CENTER :X
   RT 30
   REPEAT 3 [FRACTALT :X :Y RT 120]
   PU HOME PD
END

TO CENTER :X
   PU
   HT
   SETXY - :X/2 (-:X/4)
   PD
   FULLSCREEN
END

TO FRACTALT :X :Y
   IF :Y = 0 THEN FD :X STOP
   FRACTALT :X/ 3 :Y - 1
   LT 60
   FRACTALT :X/ 3 :Y - 1
   RT120
   FRACTALT :X/ 3 :Y - 1
   LT 60
   FRACTALT :X/ 3 :Y - 1
END
```



Reprinted from Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers' of Mathematics.

Problem: Four towns are placed at the corners of a 10 km square. A highway network is needed to link all four towns. What is the shortest network you can plan?

Answer: ≈ 28.3 km

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, October 1988.

Problem: A planner wants to link up five cities (no three collinear) by freeways. Each city must be linked to every other one. What's the least number of roads required? Roads can cross by means of overpasses, of course.

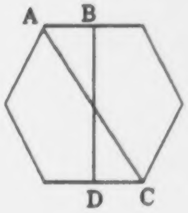
The planner then decides that overpasses are very costly. What is the fewest number of overpasses required?

Answer: 10 roads, likely with 5 crossings. It can be reduced to one crossing (with a rather large overpass!)

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, October 1988.

Problem: A farmer chooses chicken wire that has perfect hexagonal design, with each side of the hexagon measuring 2 cm. Can chicks with a minimum dimension of 0.041 m sneak through the hexagons?

Answer: No



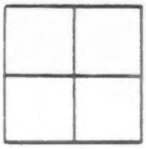
AB = 4cm
CD ≈ 0.03464cm

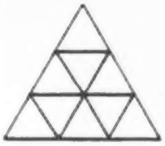
Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1986.

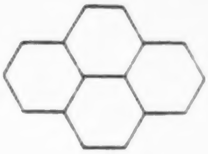
Activity: Have students explore the possibility for creating tessellations with polygons. Technically speaking, tessellation is the process of completely covering a plane by a set of non-overlapping polygonal regions. For example, through exploration, students may discover that of the regular polygons, only the square (vertex angle 90°), the equilateral triangle (vertex 60°), and the regular hexagon (vertex angle 120°) can be used alone to create a tessellated plane.

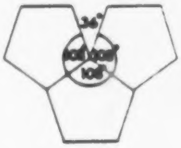
What is the governing factor? Why is a regular pentagon not a suitable figure for tessellating when used alone?

(The vertex angle must divide 360° integrally. The regular pentagon, with a vertex angle of 108°, can therefore not be used.)









Adapted from "A Tiny Treasure of Tessellations," Imaginative Ideas for the Teacher of Mathematics Grades K–12: Ranucci's Reservoir copyright 1988 by the National Council of Teachers of Mathematics.

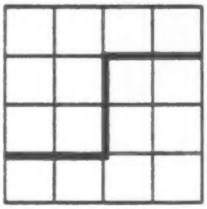
Problem: At the beginning of every meeting, each member of a club shakes hands with every other member exactly once. If six members attend a meeting, how many handshakes are made?

Answer: 15

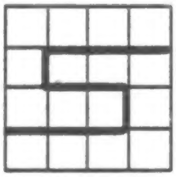
Solution suggestion: Draw a sketch in the form of a hexagon of the member shaking hands.

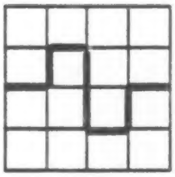
Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1986.

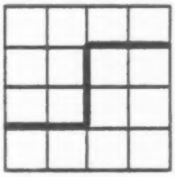
Activity: Have students explore how many different paths they can find that cut a 4 by 4 array of squares into two congruent parts. They must stay on grid lines. One path is shown.

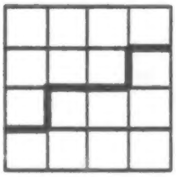


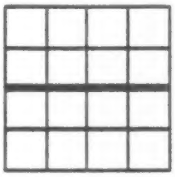
Answers:

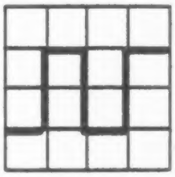












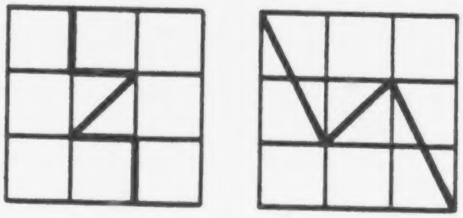
Once the paths have been created, have students examine them. For example, what features or properties are shared by all solutions? (All paths must have point symmetry about the centrepoint and hence must pass through that point.)

Activity: Follow up the previous activity with the question. "Can a 3 by 3 square array be cut into two congruent halves cutting only on grid lines?"

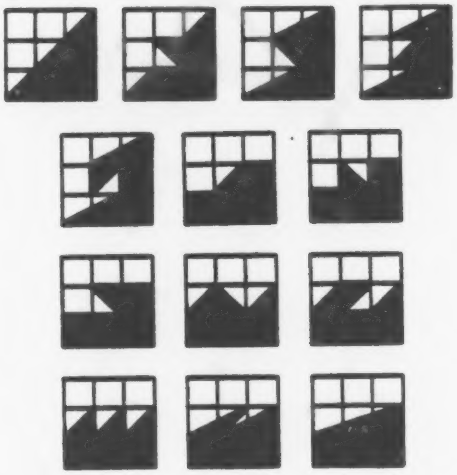
Answer: No, because maintaining symmetry about the centrepoint is not possible under such conditions.

Activity (**Extension**): A 3 by 3 array can be cut into two congruent halves by putting along a polygonal path with vertices located on the grid. Two examples are shown; try to find the rest.

Examples:



Answers:



RESOURCE CORRELATION

MMW3:	pp.
AM11:	pp.
MM3:	pp. 253–254
MPMM3:	pp.

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that within closed two-dimensional figures, there are relationships amongst the sides and angles.

Students will be expected to:

1.2 determine patterns to develop a formula for perimeter, which is the distance around a figure or object
  1.2.1 calculate the perimeter of any regular polygon
  1.2.2 solve problems involving the perimeter of regular polygons
  1.2.3 determine that the ratio between the circumference and diameter of a circle is constant
  1.2.4 solve problems involving the circumference of circles

COMMENTS

Students are often expected to know "perimeter", however, they may only know perimeter as $2l + 2w. = P$ and not be able to relate perimeter to circumference. Hence, it is important that the formula be developed. Students should build an understanding of, and be able to describe, perimeter.

It is important here that students participate in activities that go beyond the memorization of a series of formulas. Students should have the knowledge to be able to determine how to calculate the perimeter of any polygon without the use of a formula.

Note: In discussing π in 1.2.3 students should measure the circumference and diameter of a number of circles to see that the ratio is approximately the same. See the Mathematics 10 Number Systems portion of Mathematics 10/13/14 Interim Teacher Resource Manual for more activities dealing with π.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Polygons – Have students measure the distance around several physical objects such as a door, desk top, window, prior to measuring polygons. This provides the student with an experience that demonstrates that "perimeter" not only exists in mathematics. It is important to also discuss at this point the appropriate measuring devices to be used, depending on the accuracy required.

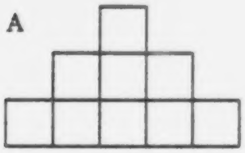
Activity: To have students realize that figures that have the same perimeter are not necessarily the same figure, have students make a figure, given a length of string (the same length to several students) – the perimeter is the same. Questions that could be explored at this point: Does the size of the perimeter determine the number of sides in a polygon?; Does the perimeter affect the degree measure of the polygon?; If two figures are joined, will the perimeter of the new figure be the perimeter of figure 1 plus the perimeter of figure 2 (have students join their figure with another students' and measure the perimeters to discover if this is so).

Problem: You know that the perimeter of a certain rectangle measures 22 cm. If its length and width each measure a whole number in centimetres, how many different areas (in square centimetres) are possible for this rectangle? Answer: 5.

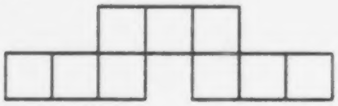
Problem: Figures A and B below are made up of congruent squares. If the perimeter of Figure A is 48 cm., what is the perimeter of Figure B?

Answer: 60 cm

A



B



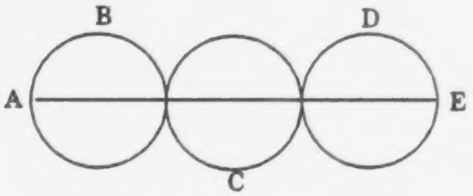
Activity: Circles – Have students discuss the possible ways of measuring the distance around circles. Have students use strings, and possibly measuring tapes, to measure the distance around circular objects – this will help to reinforce how important it is to have a formula for the circumference of a circle.

Activity: To assist in the understanding of π, have students measure the diameters and circumferences of a set of circles and have them notice that the ratio of these two attributes is approximately 3.14 in each case. Students could fill out a chart as follows:

Circle	Diameter	Circumference	C/D
A			
B			
C			
D			

Problem: The three touching circles shown are identical. Line segment AE = 42 cm long. How long is the curved path ABCDE?

Answer: 66 cm.



Activity: Have students describe "perimeter" in their own words.

TECHNOLOGY INTEGRATION

Spreadsheet Example: AM11: pp. 477–478

ELECTIVE SUGGESTIONS

Polygons – A discussion of "isoperimetric" – polygons that have the same perimeter.

Problem: Metric paper sizes have the property that a sheet of paper cut in half gives two smaller sheets of the same shape as the original sheet. Find the ratio of the sides of the sheets.

Answer: $l:w :: \sqrt{2}:1$

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Problem 1.2.4: Which is greater: the distance around the lip of a can of three tennis balls or the height of the can?

Answer: Distance around the lip.

[The height is equivalent to 3 diameters; the distance around the lip is equivalent to π diameters.]

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1986.

RESOURCE CORRELATION

MMW3:	pp. 344–346
AM11:	pp. 170–173
MM3:	pp. 222–229
MPMM3:	pp. 344–346

CONCEPT 1 (continued)

Students will be expected to **demonstrate an understanding that within closed two-dimensional figures, there are relationships amongst the sides and angles.**

Students will be expected to:

- **1.3 determine patterns to develop a formula for area that is a measure of the number of non-overlapping square units which can be enclosed within a figure**
  - **1.3.1 calculate the area of any regular polygon**
  - **1.3.2 calculate the areas of irregular regions that can be subdivided into regular polygons**
  - **1.3.3 solve problems involving the area of polygons**
  - **1.3.4 determine what figure encloses the largest area for a given perimeter**
  - **1.3.5 calculate the area of a circle**

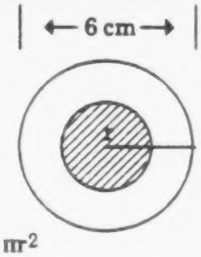
COMMENTS

Once again, students are not only expected to apply the formulas for the area of polygons but to understand and describe area. Student are more likely to develop a better understanding of areas if they have the opportunity to see the "interconnectedness" between finding the areas of different polygons.

Note: 1.3.4 is a good problem solving expectation (circle).

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: This problem provides an opportunity to review area. Find r such that the areas of the washer and the hole are equal.



Area of the hole: πr^2

Area of the washer: $9\pi - \pi r^2$

Possible solution: $\pi r^2 = 9\pi - \pi r^2$

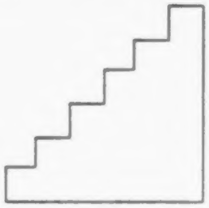
$2r^2 = 9$

$r = 3/\sqrt{2}$ or $r \approx 2.12$

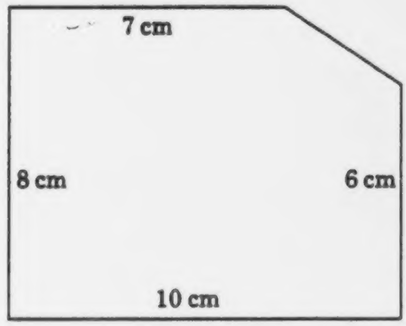
Activity: Polygons - Graph paper and geoboards are useful in building an understanding of area. The need for an indirect measure of area (hence the need for formulas) will become evident when area becomes difficult to "count".

Problem: Find the area of the figure below. The line segments for the "step" formation meet at right angles and are of 1 cm length.

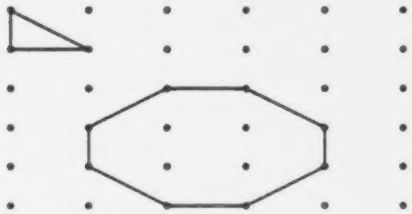
Answer: 21 cm^2



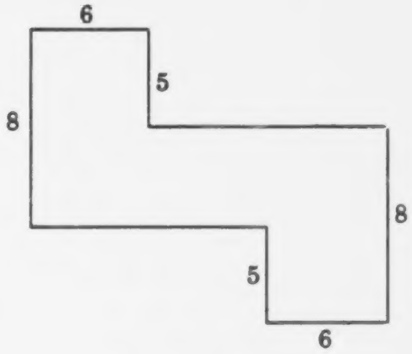
Problem: Find the area: **Answer:** 77 cm^2



Problem: The area of the triangle is 2 cm^2. What is the area of the octagon? **Answer:** 28 cm^2



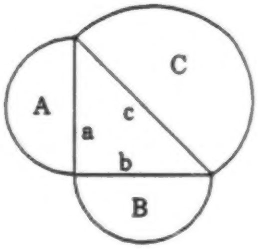
Problem: If the area of this figure is 108 square units, then what is its perimeter? **Answer:** 58



Activity: Circle – Have students cut circles into portions and form a parallelogram and then attempt to measure the area – once again due to the inaccuracy, it is important to have a formula to find the area of a circle.

Problem: Pythagoras showed that for a right triangle $a^2 + b^2 = c^2$. Is it true that for a right triangle the area of the semicircles are related by Area A + Area B = Area C?

Answer: Yes.

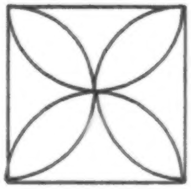


Activity: Have students write a definition of area. Have students describe the difference between area and perimeter.

Problem: You know that the perimeter of a certain rectangle measures 22 cm. If its length and width each measure a whole number in centimetres, how many different areas (in square centimetres) are possible for this rectangle?

Answer: 5.

Problem: Given a square with sides equal to 8, the midpoint of each side of the square is the centre of the circle with radius equal to one-half the side. Four semicircles are drawn. What is the area of the resulting shamrock?



Answer: $A = 32\pi - 64$

One solution: Work with a single quadrant to find the area of one-half of one leaf. Multiply by eight.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, May 1987.

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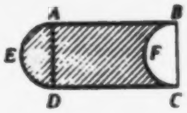
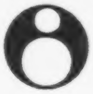
Problem: *ABCD* is a rectangle with $AB = 2AD$. *AD* and *BC* are diameters of semicircles *AED* and *BFC*. If $AD = 6$, what is number of square units in the area of *ABFCDE*?



Answer: 72 square units

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1987.

Problem: Two small circles with radii 2 and 3 are externally tangent. A third larger circle is circumscribed about these two as shown. Find the ratio of the area of the smallest circle to the area of the shaded region. Express your answer as a common fraction.



Answer: 1:3 or ⅓

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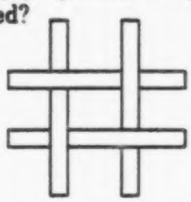
Problem: Four rectangular paper strips of length 10 and width 1 are put flat on a table and overlap perpendicularly as shown. How much area of the table is covered?

(A) 36 (B) 40 (C) 44
(D) 96 (E) 100



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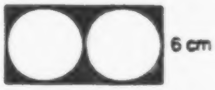
Problem: Find the area of the shaded region.



Answer: Area of shaded region ≈ 15.48 cm^2

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1988.

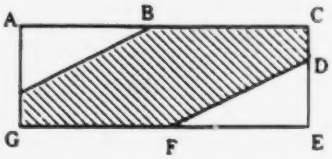
Problem: *ABCD* is a square. *A* is folded onto the midpoint of *AB*. The perimeter of the smaller figure formed is 25. Find the area of *ABCD*.

Answer: Area = 100 sq. units.

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1988.

Problem: If B, D, F, and H are the midpoints of the respective sides of the rectangle $ACEG$, find the area of hexagon $BCDFGH$ where $BC = 9$ and $CD = 4$.

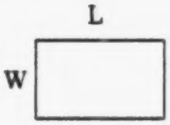
Answer: $A = 18 \times 8 - (\frac{1}{2} \times 4 \times 9)$
$= 18 \times 8 - 4 \times 9$
$= 144 - 36$
$= 108$ square units



Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1986.

Problem: The dimensions of a rectangular garden with a perimeter of 100m must be greater than 20m. What is the greatest difference between the areas of two such gardens?

Answer: (assuming integral dimensions) : $25^2 - 29 \times 21$
$= 625 - 609$
$= 16\text{m}^2$



$P = 2(L + W)$
and $P = 100\text{m}$
$\therefore L + W = 50\text{m}$
and $\therefore 20 < L < 30$ and $20 < W < 30$

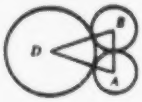
This problem lends itself to a graphing solution to determine minimum and maximum areas under the conditions stated.

Extension: If a circular garden, with circumference 200m was also permitted, what is the difference in area between the circular garden and the rectangular garden of smaller area?

Answer: Difference $= 795.8\text{m}^2 - 609\text{m}^2 = 186.8\text{m}^2$

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1986.

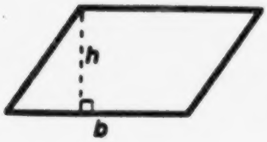
Problem: If C is the circumference and if
$C_A = 8\pi$, $C_B = 8\pi$, $C_D = 12\pi$, what is the area of triangle ABD?

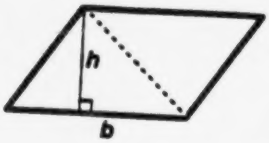


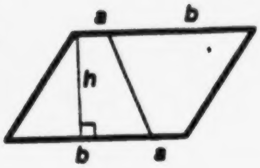
Answer: $A = 8\sqrt{21}$

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1987.

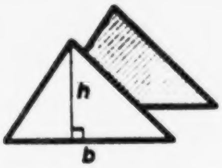
Activity: Have the students explore how the area of figures such as the triangle, rectangle and trapezoid can be found, given that the area of a parallelogram is determined by multiplying the base and the height.



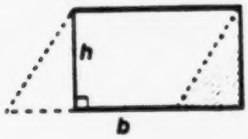




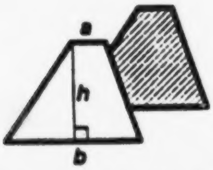
Answers: One form of solution in each case:



A = ________
(Triangle)



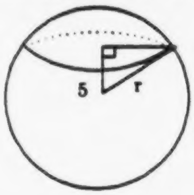
A = ________
(Rectangle)



A = ________
(Trapezoid)

Figures reprinted from "Geometry Formulas," Teaching With Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

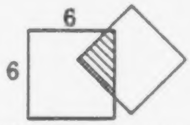
Problem: A plane passes 5 cm from the centre of a sphere. If the area of the circle of intersection is 144 π, find the radius of the sphere.



Answer: 13 cm

Reprint from Mathematics Teacher, National Council of Teachers' of Mathematics, December 1986.

Problem: Two congruent 6 cm × 6 cm squares overlap as shown. A vertex of one square is at the centre of the other square. What is the largest possible value for the shaded area? (The top square is movable about its fixed vertex.)

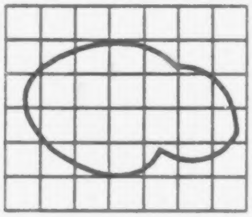


Answer: 9cm^2 (no matter what the position of the top square.)

From "Creative Geometry Problems Can Lead to Creative Problem Solvers," in Learning and Teaching Geometry, K–12, 1987 NCTM Yearbook.

Activity: Approximating area. To strengthen the concept of area, on squared paper have students draw (or supply to the students) irregular but smooth, closed figures. For each figure, have the students determine an upper bound on the area by counting all the squares required to completely cover the area enclosed in each figure. Next determine the lower bound on the area by counting all the squares that are completely enclosed by each figure. Transform these upper and lower bound counts into areas, then determine the average as an approximation of the area of each figure.

Example sketch:



TECHNOLOGY INTEGRATION

Spreadsheet Example: AM11: pp. 477–478

Polygons – A discussion of "isoperimetric" – polygons that have the same perimeter.

See "Problem-Solving Techniques with Microcomputers," Mathematics Teacher, November 1986, for a discussion of and BASIC language programs dealing with the minimum-maximum area problem.

RESOURCE CORRELATION

MMW3:	pp. 347–349
AM11:	pp. 174–179
MM3:	pp. 230–236
MPMM3:	pp. 347–349

CONCEPT 2

Students will be expected to demonstrate an understanding that within three-dimensional figures, there are relations amongst the sides, edges and vertices.

Students will be expected to:

- 2.1 determine patterns which can be used to discover relationships pertaining to three-dimensional figures
  - 2.1.1 determine the relationship between the number of edges, vertices and faces in a simple polyhedron (Euler's formula $V - E + F = 2$)
  - 2.1.2 determine the relationship between the number of faces and diagonals in a polyhedron

COMMENTS

Take time in class to discuss methods of drawing three-dimensional solids. This may help develop students' understanding of the properties of these figures. Graph paper is a useful medium, as it highlights congruent shapes and parallel and perpendicular edges.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Polyhedra - An examination of polyhedra that exist in the student's life should be used as an introduction. e.g. a cereal box or cracker box are examples of polyhedra and then separated at the "glue" spots to understand what a net is. Encourage students to examine similarities and differences of the representation of different nets with different polyhedra. Have students reconstruct the boxes in order to see "how" a net produces a polyhedron. From this point, students should be involved in investigating different polyhedron to examine the relationship between the edges, faces, and vertices - this helps and further reinforces patterns in mathematics.

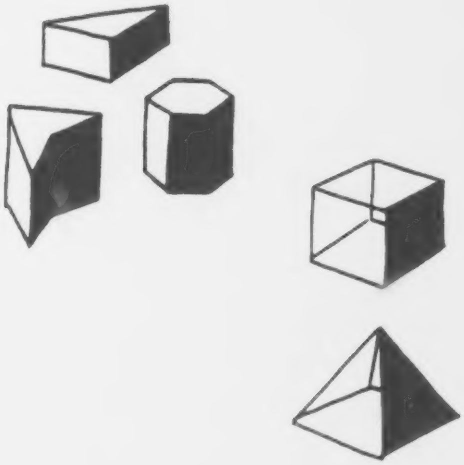
Problem: A wooden cube that measures ten cm along each edge is painted red. The painted cube is then cut into two–cm cubes. How many of the two–cm cubes do not have red paint on any face? Students should make a model (sugar cubes are good for this) using smaller measurements and then examine how many cubes would not have any red paint on any face initially and then expand it to a larger cube. A pattern should occur.

Answer: 27

Activity: Polyhedron – Using the discussion and nets – surface becomes an extension of this discussion. Have students measure the area of each portion of net to discover the concept of surface area.

Activity: Have students construct polyhedron of specific surface area, using nets – this enhances the discussion of polyhedron and surface area, i.e. are polyhedron with the same surface area necessarily the same figure?

Activity: Make sure students understand the meaning of vertex (plural vertices), edge, and face when describing three-dimensional objects. In the cube, for example, there are eight corners (points) or vertices. There are twelve edges and six faces. An Egyptian pyramid has five vertices, five faces, and eight edges. Some three-dimensional objects are sketched below. The table summarizes the appropriate entries. Notice that the entry in the E column seems to be 2 less than the sum of the entries in columns F and V. This may be indicated by the formula F + V = E + 2. It is known as Euler's Formula.



	F	V	E
A	5	6	9
B	6	8	12
C	8	12	18

Euler's Formula lends itself to the following activity. Collect a series of wooden blocks in a variety of forms and shapes. The only requirement is that the faces of the blocks be flat. Have students count the number of faces, vertices, and edges of each block and work in pairs to confirm each other's bookkeeping. Demonstrate the Euler relation. Have students consider these questions. What is the fewest number of faces which may meet to form a vertex? What is the fewest number of colours which may be used to distinguish the six faces of a cube? What is the fewest number of colours which may be used to distinguish the five faces of an Egyptian pyramid? Is it possible to cut through a wooden cube in such a manner as to get a cross section with six edges? If a cube is 9 cm long, how many times must it be sliced to form 3 cm cubes? How many such small cubes will be formed? How does the flat pattern of a cube look?

Adapted from "4 Areas in the New Math," Imaginative Ideas for the Teacher of Mathematics Grades K-12: Ranucci's Reservoir, copyright 1988 by the National Council of Teachers' of Mathematics.

Activity: Distribute the nets unnamed to the students, and have them predict the solid that will be produced when folded as indicated.

Then have the students construct the cube (regular hexahedron), regular octahedron, and square based right pyramid. It may be easier and more efficient if the students work in pairs to cut out, fold and tape the solids.

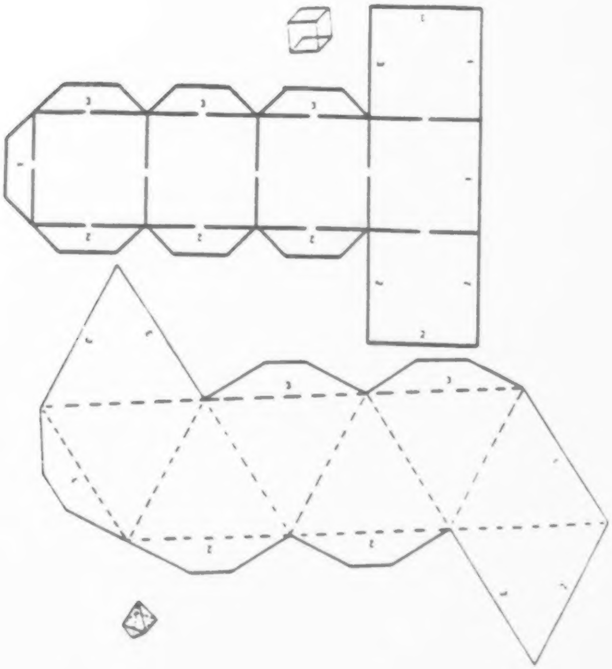
These figures can be used to help understand Euler's Formula, $V + F = E + 2$.

Elective: Construct six of the square based right pyramids shown above. Put these together to make a cube.

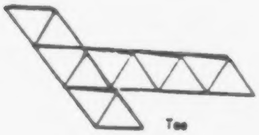
One solution: Place four pyramids, resting on their base, side-by-side to form a row. Place the remaining two pyramids on either side of one of the end pyramids in the row. The resulting form should be a "Tee". Tape the adjoining edges together, and fold inward to form a cube. (Is there a point at which all the pyramids are touching each other?)

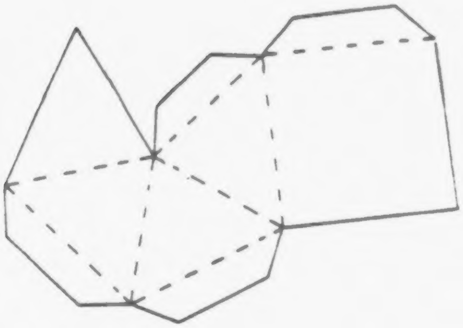
Elective: Use the nets shown to construct four copies of the solid. (What shape will it form? - Tetrahedron.) Then put these four together to form a regular tetrahedron.

One solution: Place the four tetrahedra together side-by-side on their large bases to form a parallelogram when viewed from overhead. Tape and fold inward.

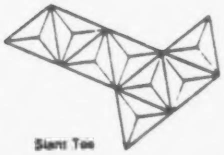


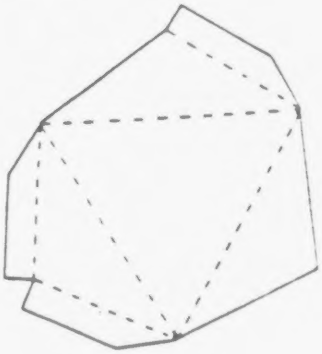
Put six of these together to make a "Tee".
Wrap the Tee inward to make a cube.
Wrap the Tee around the cube to make a Rhombic Dodecahedron





Tape eight of these together to make a "Slant Tee"
Wrap the Tee inward to make two tetrahedra
Wrap the Tee around the octahedron to make a Rhombic Dodecahedron





Figures reprinted from Mathematics in School, January 1989, pp. 19-21.

Activity: Give students a set of polyhedron models (diagrams would also work) and ask then to fill in the following chart.
(Thanks to L. Hovdebo, Sir Winston Churchill High School)

Polyhedron	Number of Faces	Number of Diagonals

Now, have students describe the relationship that exists between the number of faces and the number of diagonals. Extend the question to include a polyhedron not used, will this relationship always be the same?

Activity: To illustrate that Euler's Formula is true for all polyhedra, have students "cut" the corners off the edges of the cube (or any other polyhedron) and examine the number of edges, faces and vertices. Does the relationship still exist? Ask students to see if they can find a case where this relationship doesn't exist. Does this relationship work with a sphere?

(Thanks to B. Andrew, Three Hills School)

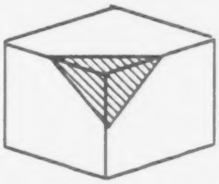
ELECTIVE SUGGESTIONS

Problem: A cube has edges of length 10cm. If a fly lands on a vertex and then walks only along the edges, what is the greatest distance the fly could walk before coming to a vertex a second time and without retracing an edge?

Answer: 80cm

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, April 1986.

Problem: Which polygons can be produced as cross-sections of a cube? This cross-section produces an isosceles triangle



Answer: Equilateral, isosceles and scalene triangles; square; rectangle; trapezoid; pentagon; and hexagon.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, February 1987.

RESOURCE CORRELATION

MMW3:
AM11:
MM3: pp. 255–257
MPMM3:

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that within three-dimensional figures, there are relationships amongst the sides, edges and vertices.

Students will be expected to:

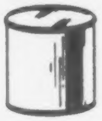
- 2.2 determine patterns to develop a formula for the surface of a rectangular solid, cone, sphere, cylinder, and pyramid
  - 2.2.1 calculated the surface area of a rectangular solid, cone, sphere, cylinder, and pyramid
  - 2.2.2 calculate the surface area of irregular solids that can be subdivided
  - 2.2.3 solve problems involving surface area of regular and irregular solids

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Polvhedron - Using the discussion and nets - surface becomes an extension of this discussion. Have students measure the area of each portion of net to discover the concept of surface area.

Activity: Have students construct polyhedron of specific surface area, using nets - this enhances the discussion of polyhedron and surface area, i.e., are polyhedron with the same surface area necessarily the same figure?

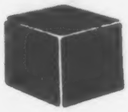
Problem: The most economical tin can to manufacture is one that is a right circular cylinder with height equal to twice the radius. One such can has a volume of 54 cm^3. What is the total surface areas of this can in cm^2?



Answer: 54π cm^2

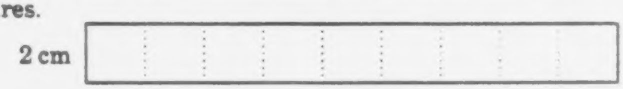
Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1988.

Problem: What is the shortest strip of paper 2 cm wide and black on only one side that can be folded (with no cutting) to form a 2-cm cube that is completely black on the outside.



Answer: 18 cm

Fold the 2-cm strip eight times to produce nine 2- by 2-cm squares.



Fold the middle square diagonally. A 2-cm cube that is completely black on the outside can now be formed.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, April 1967.

TECHNOLOGY INTEGRATION

Spreadsheet example: AM11: pp. 477, 479–480

ELECTIVE SUGGESTIONS

Problem: Given a cylinder of height 6 cm and base radius 2 cm, demonstrate that a spider can go from any point on the surface to any other point of the surface along a path of total length less than 9 cm. Use concrete visual aids to help if necessary.

Reprinted from delta-K, April 1989, "Mathematics for Gifted Students," p. 44.

RESOURCE CORRELATION

MMW3:	pp. 350–352
AM11:	pp. 180–185, 191 (extension)
MM3:	pp. 237–244
MPMM3:	pp. 350–352

CONCEPT 2 (continued)

Students will be expected to demonstrate an understanding that within three-dimensional figures, there are relationships amongst the sides, edges and vertices.

Students will be expected to:

2.3 determine patterns to develop a formula for the volume of regular solids (area of base times height)
  2.3.1 calculate the volume of regular solids (area of base times height)
  2.3.2 calculate the volume of irregular solids that can be subdivided
  2.3.3 solve problems involving the volume of regular and irregular solids
  2.3.4 determine what figure encloses the greatest volume for a given surface area

PROCESS/PROBLEM-SOLVING CONTEXT

Problem: Eight steel balls, each with a radius 1 cm, are melted down and reshaped into one large ball. What is its radius?

Answer: $r = 2$

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, May 1988.

Problem: A swimming pool whose surface forms a rectangle measures 25 m long by 15 m wide. The pool is 2 m deep at the shallow end, and the depth increases at a constant rate to 4 m at the other end. How many liters of water will the pool hold.

Answer: 1 125 000 *l*

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, September 1988.

Problem: In a can of tennis balls that is exactly three balls high, which is greater:: the volume of the balls or the volume of the air around the balls?

Answer: The volumes are the same.

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1986.

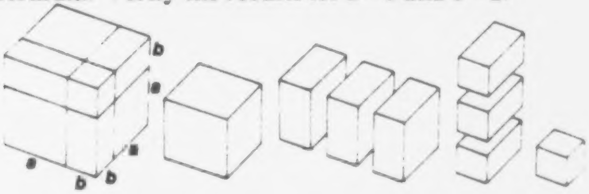
Activity: Use the science lab to measure the dimensions of prisms and cylinders and then measure the amount of liquid that the polyhedron will hold – does this amount equal the volume formula calculation? Have students identify different prisms and cylinders that have equal volumes and examine the dimensions.

Activity: Have students examine the differences and similarities between surface area and volume, i.e., if two figures have equal surface areas, do the necessarily have the same volumes?

Problem: Mr. Brook wants to build a container with a volume of 64 cm^3. He can build it either in the shape of a cube or of a rectangular prism. The rectangular prism would have to be 8 cm high and 4 cm wide. Which would have the smallest surface area, and by how much.

Answer: cube, 16 m^2

Problem: Use the volumes of the prisms formed to completed this formula. Verify the results for a = 5 and b = 2.



$(a + b)^3 =$ _____ + _____ + _____ + _____

Reprinted from "Bet you can't . . .," Teaching with Student Math Notes, copyright 1987 by the National Council of Teachers of Mathematics.

Estimation Activity: Estimate the dimensions and use theses formulas to estimate the volume of each container.

Materials

1. Formulas for volume:
   $V = Bh$, or $V = lwh$ (rectangular prism) $V = \pi r^2 h$ (cylinder)
   $V = \frac{4}{3} \pi r^3$ (sphere) $V = \frac{1}{3} Bh$ (pyramid or cone)

2. Pencils, paper, string and scissors. Do not permit any rulers or any visual access to standard units, such as millimetre, centimetre or metre.

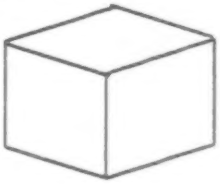
3. A variety of commonly available container,such as—
   Tennis ball can
   Drinking cup
   Desk drawer
   Waste basket
   Cereal boxes
   Cardboard box for duplicating paper
   Shoe box
   Grocery bag

4. A list showing the actual volume of each container (or the teacher's best estimate) to use as a check of the students' answers.

Encourage the students to round the numbers used in their computations.

Reprinted from "Estimation in Measurement," Estimation and Mental Calculation, copyright 1986 by the National Council of Teachers of Mathematics.

Problem: The areas of the sides of a rectangular box are 24, 32, and 48 cm^2. What is the volume of the box?



Answer: $V = 8\text{ cm} \times 6\text{ cm} \times 4\text{ cm}$
$= 192\text{ cm}^3$

(Note one solution: In solving for the dimension of the box, this problem requires student to set up a simple quadratic equation, for example, $2h^2 = 32$. Question, is it necessary to determine all three dimensions in order to answer the problem as stated?)

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, November 1987.

Problem: A large grain storage bin in the shape of a right cylinder has a diameter of 15 m and a height of 40 m. If a bin of equal volume were built to the same height, but with a square base, what would be the length of a side of the base?

Answer: 7.5 m

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1988.

TECHNOLOGY INTEGRATION

Spreadsheet example: AM11: pp. 477, 479–480

See "A computer-graphing-based Approach to Solving Inequalities," Mathematics Teacher, May 1989, for a discussion of how a graphing calculator or computer graphing program such as Master Grapher may be used to help clarify and solve variable volume problems (for example: maximum volume, less than or grater than a given volume, etc.)

ELECTIVE SUGGESTIONS

Problem: Jean had two cylindrical cans of sliced fruit. The label on the smaller one listed the volume as 513 cm^3. The label on the larger can was destroyed. Jean noticed that the smaller can had the same diameter as the larger, but that the larger was one-third taller. What was the volume of the larger can?

Answer: 683 cm^3

Reprinted from Mathematics Teacher, National Council of Teachers' of Mathematics, January 1988.

RESOURCE CORRELATION

MMW3:	pp. 353–362
AM11:	pp. 186–189, 190 (extension)
MM3:	pp. 245–252
MPMM3:	pp. 353–362

TRIGONOMETRY
PROGRAM EMPHASIS - 13%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that two triangles are similar if corresponding angles are equal and the lengths of the corresponding sides are proportional.	Students will be expected to identify and express the similarity correspondences between similar triangles.	
	Students will be expected to use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides.	Students will be expected to derive the properties of similar triangles experimentally. Students will be expected to solve problems involving the use of the properties of similar triangles. Students will be expected to use the properties of similar triangles to investigate the sine, cosine, and tangent ratios in similar right triangles.
Students will be expected to demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of sides of a right triangle.	Students will be expected to find the sine, cosine and tangent ratios, and the measures of the acute angles within right triangles, given the measures of any two sides.	Students will be expected to find the sine, cosine and tangent ratios of any acute angle, using a calculator. Students will be expected to find the measure of an acute angle given the value of one of the trigonometric ratios of the angle, using a calculator. Students will be expected to solve problems in which an acute angle measure in a right triangle must be found.
	Students will be expected to find the measure of a side of a right triangle given the measure of an acute angle and the length of a side.	Students will be expected to solve problems in which the length of a side of a right triangle is unknown.

CONCEPT 1

Students will be expected to demonstrate an understanding that two triangles are similar if corresponding angles are equal and the lengths of the corresponding sides are proportional.

Students will be expected to:

1.1 identify and express the similarity correspondences between similar triangles
  1.1 identify and express the similarity correspondences between similar triangles
  1.2 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides
    1.2.1 derive the properties of similar triangles experimentally
    1.2.2 solve problems involving the use of the properties of similar triangles
    1.2.3 use the properties of similar triangles to investigate the sine, cosine and tangent ratios in similar right triangles

COMMENTS

Students were introduced to the concept of "similarity" in Grade 7 and again in Grade 10. The study at these times included recognizing similar and congruent figures. The concept of similar triangles can be related to everyday life. This can become the basis for an outside-of-classroom project.

This is the first time that students in Math 13-23-33 will be dealing with trigonometry.

Define similar triangles as triangles which have corresponding angles of equal measure.

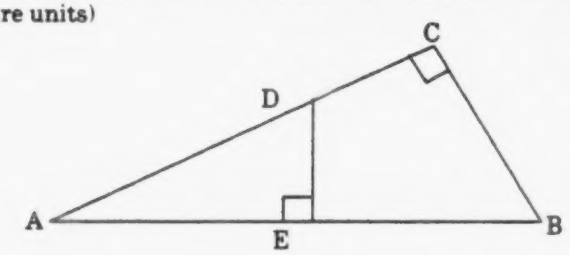
Note: The study of reciprocal ratios is included in Math 33.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: In order to introduce similar triangles, give students the measures of three angles and ask them to draw any triangle with those three measures. Have them label the triangle and then collect the measures of the sides of the triangles - the sides should be proportionate. From here, have students construct their own pair of similar triangles - ensuring that the angles are equal and that the sides are proportionate. This leads into a discussion on the way the similarity relationship is expressed.

Activity: Draw pairs of similar triangles. Measure the lengths of the sides. Find the 3 ratios between the lengths of the sides of each triangle and compare. Discuss why the ratios will be equal. Mention that similarity can be defined as "a transformation of simultude," i.e., one figure is a magnification or reduction of the other." Discuss whether or not the relationships between ratios of corresponding sides will continue for other pairs of similar polygons.

Problem: In the figure, ∡C is a right angle, DE ⊥ AB, AE = 6, EB = 7, and BC = 5. What is the area of EBCD? (Answer: 22.5 square units)



Problem: See Problem Solving in Mathematics: Focus for the Future, p. 58, "the Telephone Pole Problem", solution #2; p. 70, #1.

Activity: Have each student carefully draw one or two of the following 9 right triangles:

5° - 85°, 10° - 80°, 10° - 80°, 15° - 75° ... 45° - 45°

Measure the sides and find the sine, cosine, and tangent ratios. Use calculators to find ratios to two decimal places. Make a chart showing the ratios for each of the angles drawn. Discuss some of the characteristics of the list (for example, sine and tangent ratios increase as angles increase, cosine ratios decrease, tan 45° = 1, sin 45° = cos 45°, range of sin, cos, tan for $\theta < 90°$).

Activity: Have students work in a group and have them work through a "Similar Triangles Project". Provide students with a group of triangles where there are six pairs of similar triangles. Provide the following definition and directions:

Definition: Similar triangles are any triangles that have all angles congruent. The sides do not necessarily have to be the same length.

Directions:

- Name one pair of triangles that are similar according to the definition given above.
- Measure the lengths of the sides that correspond and write it as a ratio.
- Repeat this procedure with all other pairs of similar triangles.
- Record the information in a chart such as:

Similar Triangles	Corresponding Side	Ratio	Similar Triangles	Corresponding Side	Ratio

What conclusion can be drawn about the relationship of the sides in similar triangles?

(Thanks to B. Andrew, Three Hills School.)

TECHNOLOGY INTEGRATION

Software:
The Geometric preSupposer: Points and Lines and The Geometric Supposer: Triangles (Sunburst) may both be used for exploratory activities.

Video:
Trigonometric Functions I (TV Ontario): Trigonometric Ratios.

ELECTIVE SUGGESTIONS

Extension of the area of EBCD problem on page 204. Consider the following theorem: If two figures are similar with ratio of similitude r, then any two corresponding areas are in the ratio r^2 How could this theorem have been applied to determine the area of EBCD?

RESOURCE CORRELATION

MMW3:	pp. 398–406, 409–411
AM11:	pp. 216–217, 236–240, 244
MM3:	pp. 282–290
MPMM3:	pp. 398–406, 409–411

CONCEPT 2

Students will be expected to demonstrate that the since, cosine and tangent ratios of a given angle are particular ratios between pairs of sides of a right triangle.

Students will be expected to:

- 2.1 find the sine, cosine and tangent ratios, and the measures of the acute angles within right triangles, given the measures of any two sides
  - 2.1.1 find the sine, cosine and tangent ratios of any acute angle using a calculator
  - 2.1.2 find the measure of an acute angle given the value of one of the trigonometric ratios of the angle, using a calculator
  - 2.1.3 solve problems in which an acute angle measure in a right triangle must be found.

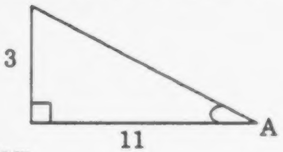
COMMENTS

Students should be taught to use a scientific calculator to find the sine, cosine and tangent ratios. Angles should be whole degrees only. Example: tan 56 ≈ 1.4826. Students should also be familiar with the following type of question. Example: $\sin\theta \approx 0.4567$

$$\theta \approx 27°$$

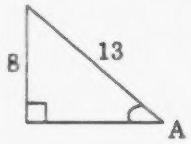
Students should use a calculator and be reminded that they are finding a specific ratio. They should always note which ratio is being found. Write the ratio to a maximum of 4 decimal places.

Examples:



$\tan \measuredangle A = 3/11$

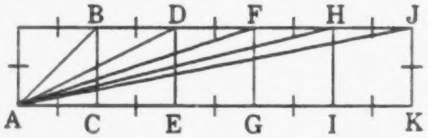
≈ 0.2727



Find the measure of ∡ A, to the nearest degree.

PROCESS/PROBLEM-SOLVING CONTEXT

Problem:



Find the measures of the 3 trigonometric ratios of ∡ A and the measure of ∡A in triangles ABC, ADE, ADG, AHI, and AJK. Can you generalize the ratios and predict the measure of ∡A in the tenth triangle?

Problem: As $\tan\theta$ increases, $\cos\theta$ decreases. What is the measure of the angle where they are equal? What are the measures of the sides of a triangle in which $\tan\theta = \cos\theta$?

Activity: Ask students to develop processes whereby in any right triangle, given any two sides or one side and one non–right angle, they could find all of the other measures of the triangle. Remind them that they know the Pythagorean Theorem. Have them develop quick checks that their answers are reasonable, i.e., what must be the sum of the angles? What must always be true about the sum of the lengths of the two shorter sides in the triangle?

Activity: Have students draw right triangles randomly, measure the lengths of any two sides or one angle and the length of one side, then calculate the unknown measures. Check the calculations by measurement

Activity: Have students develop a test whereby they can check an angle for "rightness" without using a protractor.

Problem: A parking lot beside a curling rink has room for 20 cars parked perpendicular to the building. In order to incre the width of the driveway, it was decided to have the cars park a a 75° angle to the building. Assuming that each parking space measures 3 m by 7 m, how many cars can be parked now?

Answer: 19.

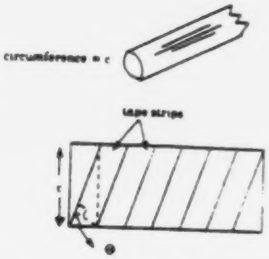
TECHNOLOGY INTEGRATION

Spreadsheets may be effectively used here. See, for example, "Integrating Spreadsheets into the Mathematics Classroom, Mathematics Teacher, November, 1988.

See also: AM11: pp. 481–482.

ELECTIVE SUGGESTIONS

Problem: Consider a baseball bat handle slightly modified so that it is cylindrical in shape. It can be taped so that there is no overlap and no gaps, that is, no wood is showing. If the circumference of the handle is 8 cm and the width of the tape is 2.5 cm, find the angle, relative to the line of the handle, at which the tape should be wrapped for no overlap and no gaps to occur. See the diagram as follows:



Find θ

Answer: 72° to the nearest degree.
Extension: if c = circumference of the handle and w = width of the tape, express cos θ and θ in terms of w and c.

Adapted with permission from Imaginative Ideas for the Teacher of Mathematics Grades 11-12: Ranucci's Reservoir, copyright 1988 by the National Council of Teachers of Mathematics

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RESOURCE CORRELATION

MMW3:	pp. 406–408, 411–413, 415, 419–423
AM11:	pp.240-246, 248–249, 252-255
MM3:	pp.291-295
MPMM3:	pp. 406–408, 411–413, 415, 419–423

CONCEPT 2

Students will be expected to demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of sides of a right triangle.

Students will be expected to:

2.2. find the measure of a side of a right triangle given the measure of an acute angle and the length of a side

  2.2.1 solve problems in which the length of a side of a right triangle is unknown

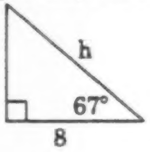
COMMENTS

Students should learn to solve problems which can be diagrammed using right triangles. They should understand and use terms such as inclination, angle of depression and angle of elevation. They should be taught the value of diagramming problem situations, and taught about the level of accuracy required.

Examples:

In ABC, ∡ C = 90°, a = 5, b = 8. Find the measure of ∡ B to the nearest degree.

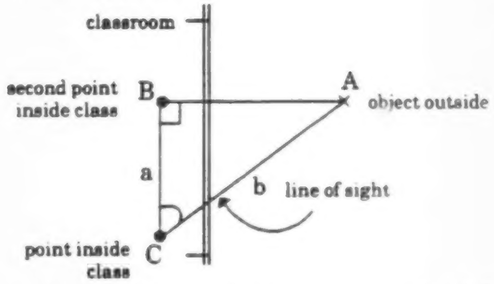
Find the length of h, to one decimal.



In XYZ, ∡ X = 90°, ∡ Y = 43°, x = 20. Find the length of z.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: If you have a classroom with windows, have students solve the following problem:



Determine the length a, and the measure of ∡BCA, and use these value to determine the distance from C to A.

Adapted from M10 TRB, Teaching Notes, p. 45.

Problem: In a 5–12–13 right triangle, what is the length of the bisector of the larger acute angle?

Answer: $\frac{5\sqrt{13}}{3} \approx 6.0$

Activity: Trigonometry Project

Purpose:
To measure the heights and distances of various objects around the school.

Directions:
1. How to use the clinometer:
   - aim at top of the object
   - pull trigger
   - when disc becomes stationary, release trigger
   - read angle.
2. How to use protractor and plumb line:
   - aim base of protractor at top of the object
   - drop the plumb line down
   - when string becomes stationary, hold against protractor
   - read the angle.

Materials:
Clinometer or protractor and plumb line, measuring tape, chart to record findings.

Methods:
1. Measure off the distance from the base of the object to the point where you are calculating the angle from.
2. Record distance.
3. Use clinometer or protractor and plumb line to find angle.
4. Record angle.
5. After finding all required angles and distances, return to the room for final calculations.

Object	Distance from the Object	Angle
1. South Gym Height in SW corner		
2. Shop Height on NE corner		
3. Height of West Football Standard		
4. Height of Library Ceiling		

This activity may be done in groups of two or three but each individual must submit their own calculations and show their own work.

(Thanks to B. Andrew, Three Hills School)

TECHNOLOGY INTEGRATION

Spreadsheet example: AM11: pp. 481–483.

Video:
Trigonometric Functions I (TV Ontario) Solving Right Triangles.

RESOURCE CORRELATION

MMW3: pp. 406–408, 411–414, 419–423, 426–427
AM11: pp. 240–246, 248–249, 252–255
MM3: pp. 296–309
MPMM3: pp. 406–408, 411–414, 419–423, 426–427

REVERSE TEXTUAL CORRELATION
GAGE EDUCATIONAL PUBLISHING COMPANY
MATHEMATICS FOR A MODERN WORLD
(Book Three, Third Edition)

Page	Unit	Learner Expectation Number	TRM Page
2–3	Time		
4–18	Number Skills Review		
19	Algebra	1.2	135
24–25	Evaluation, Powers and Roots		
26–27	Exponent Laws		
28–30	Powers and Radicals	1.3	125
31–32	Number Systems		
33–36	Powers and Radicals	1.2	125
37–39	Algebra	1.2	135
40	Number Skills Review		
41–43	Scientific Notation		
44	Algebra	1.2	135
48–55	Polynomials: Operations		
56–57	Polynomials: Common Factoring		
58–63,65	Algebra	2.1	141
64	Polynomials: Difference of Squares Factoring		
70–83	Algebra	1.1, 2.3, 1.2	135,146
87–98	Rational Algebraic Expressions		
100–101	Coordinate Geometry: Graphing Ordered Pairs		
102–105	Linear Relations	1.5	155
106–109	Coordinate Geometry: Length and Midpoint of Line Segments		
114–116	Coordinate Geometry		
117–122	Linear Relations	1.1, 1.2, 1.3	153
123–124	Linear Relations	1.5	155
125–126	Linear Relations	1.4	153
132–135	Systems of Equations	1.1	159
136–143	Systems of Equations	1.2	161
147–156	Review of Chapters 1–8		
157–174	Earning Money		
175–192	Calculating Income Tax		
193–204	Spending Money		
206–213	Simple Interest		
214–221	Powers and Radicals	2	129
226–240	Powers and Radicals	2	129
243–262	Cost of Transportation		
263–282	Operating a Business		
283–292	Review of Chapters 9–15		
293–320	Statistics		
322–342	Geometry: Review, Angles, Circles, Pythagorean Theorem, Congruence		
344–356	Geometry	1.2, 1.3, 2.2, 2.3	181–190, 196–200
357–362	Measurement Summary and Applications		
367–378	Ratio, Proportion and Rate		
380–387	Linear Relations	1.5	155
388–392	Inverse and Combined Variation		
398–415	Trigonometry	1, 2	203–211
416–418	Trigonometry: Reciprocal Ratios		
419–423	Trigonometry	2.1.3	206
424–425	Angle Measure (Degrees, Minutes, Seconds)		
426–427	Trigonometry	2.2.1	209
431–441	Review of Chapters 16–21		

REVERSE TEXTUAL CORRELATION
MCGRAW-HILL RYERSON LIMITED
APPLIED MATHEMATICS 11
(Second Edition)

Page	Unit	Learner Expectation Number	TRM Page
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18–23	Ratio, Proportion, and Rate		
26–30	Problem Solving		
38–41	Exponent Laws		
42–51	Powers and Radicals	1	125
52–54	Operations on Radicals		
55–57	Scientific Notation		
58–59	Number Systems		
60–61	Problem Solving		
62–65	Spreadsheets in Mathematics		
72–77	Polynomials: Operations		
78–79	Polynomials: Common Factoring		
80–83	Polynomials: Evaluating Expressions		
84–87	Algebra	2.1	141
88–91	Problem Solving		
98–109	Algebra	1.1, 2.3, 1.2, 2.3	135, 146
110–111	Problem Solving		
112–113	Algebra	1.1	135
120–123	Direct and Partial Variation		
124–128	Coordinate Geometry: Slope of a Line Segment		
129–139	Linear Relations	1.1, 1.2, 1.3, 1.4	153
140–144	Linear Relations	1.5	155
145–147	Coordinate Geometry: Distance Between Two Points		
148–149	Systems of Equations	1.1	159
150–155	Systems of Equations	1.2	161
156–157	Algebra	1.1.1	135
158–162	Systems of Equations	1.2.1	161
170–191	Geometry	1.2, 1.3, 2.2, 2.3	180–190, 196–200
192–193	Problem Solving		
198–214	Geometry: Angles, Lines and Congruence		
215–217	Trigonometry	1.1, 1.2.1	203
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248–249	Trigonometry	2.2	209
250–251	Trigonometry: Reciprocal Ratios		
252–255	Trigonometry	2.2.1	209
256–257	Angle Measure (Degrees, Minutes, Seconds)		
258–259	Problem Solving		
266–301	Statistics		
310–313	Simple Interest		
314–321	Powers and Radicals	2	129
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330–331	Problem Solving		
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REVERSE TEXTUAL CORRELATION
NELSON CANADA
MATH MATTERS
(Book Three)

Page	Unit	Learner Expectation Number	TRM Page
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40–50	Review of Exponents		
51–55	Applications: Scientific Notation		
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59–62	Powers and Radicals	1.1, 1.2	125
66–71	Powers and Radicals	2.1	129
72–74	Powers and Radicals	2.2	129
75–80	Banking Services: Savings Institutions		
81–82	A Better Rate of Interest: Term Deposit		
83–84	Working Backwards: Present Value		
85	Problem Solving: Extraneous Information		
86–87	Investigations		
92–129	Review of Polynomials		
130–132	Algebra	2.1	141
133–137	Difference of Squares		
139–148	Applying Skills: Using Factoring		
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282–295	Trigonometry	1	203
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REVERSE TEXTUAL CORRELATION
ÉDITIONS DU TRÉCARRÉ
MATHÉMATIQUES POUR UN MONDE MODERNE
(livre 3)

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19	L'algebra	1.2	135
24–25	L'evaluation puissances et racines		
26–27	La loi des exposants		
28–30	Les puissances et les radicaux	1.3	125
31–32	Les ensembles de nombres		
33–36	Les puissances et les radicaux	1.2	125
37–39	L'algèbra	1.2	135
40	La revue de nombres		
41–43	La notation scientifique		
44	L'algèbra	1.2	135
48–55	La revue des polynômes		
56–57	La revue des polynômes		
58–63,65	L'algèbra	2.1	141
64	La revue des polynômes		
70–83	L'algebra	1.1, 2.3, 1.2	135, 146
87–98	Expressions rationnelles		
100–101	Géométrie analytique		
102–105	Relations linéaires	1.5	155
106–109	Géométrie analytique		
114–116	Géométrie analytique		
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125–126	Relations linéaires	1.4	153
132–135	Paires d'équations	1.1	159
136–143	Paires d'équations	1.2	161
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175–192	Calcul de l'impôt surle revenu		
193–204	Les dépenses		
206–213	Emprunt à intérêt simple		
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263–282	Gérer une entreprise		
283–292	Révision des unités 9–15		
293–320	Statistiques		
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344–356	La géométrie	1.2, 1.3, 2.2, 2.3	181–190, 196–200
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367–378	Rapports, proportions et taux		
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416–418	Rapports trigonométriques inverses		
419–423	Trigonométrie	2.1.3, 2.2.1	206–209
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426–427	Trigonométrie	2.2.1	209
431–441	Révision des unités 16–21		

MATH 24

PREPARING FOR MATH 24

Mathematics 14 is a five-credit course. Mathematics 24 is a three- or a five-credit course organized into twelve units within five themes. It is important that teachers address each of the five themes as well as provide for the specific needs of their individual classes. The required component of Mathematics 24 courses offered for three credits will be made up of attitudes, problem solving, numeration, and geometry and measurement as well as six units, including one unit from each of the five themes, plus any one other unit selected from any of the themes, chosen according to the needs and interests of the students in the class. The required component of Mathematics 24 courses offered for five credits will be made up of attitudes, problem solving, numeration, and geometry and measurement as well as ten units, including one unit from each of the five themes, plus any five other units chosen from any of the themes. The elective component in Mathematics 24 for three or five credits will be in addition to the above. The elective component may include any units not used in the required component, chosen according to the needs and interests of the students in the class.

The themes and the units within them are:

WORK
- Income
- Income Tax

BANKING
- Personal Banking
- Consumer Credit

TRANSPORTATION
- Purchasing a Vehicle
- Operating a Vehicle
- Travelling

ACCOMMODATION
- Renting and Purchasing a Home
- Mortgage and Household Insurance
- Building, Decorating and Furnishing

COST OF INDEPENDENCE
- Budgets and Getting Value for Money Spent
- Life and Health Insurance

When preparing to teach Mathematics 24, you may wish to consult with the CALM teacher. Many of the themes and units in Mathematics 24 will complement areas of study in CALM. For instance, in CALM students might discuss different approaches in applying for a job – in Mathematics 24 students would concentrate on the calculations of wages and benefits once a job has been obtained. Students enrolled in both courses would benefit from the expanded study of a topic and would see the "connectedness" between the courses. Other things you may wish to consider when selecting the units for your Mathematics 24 course are:

- availability of community resources (e.g., is there a life insurance representative that would be available to talk to the students?)

- time of year (e.g., if I chose the Income Tax unit – will students be able to do their own? – yes in the spring)

- student interests.

PROGRAM EMPHASIS

Required Content

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises of 80% of the course and contains the concepts, skills, and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to the problem solving and the use of technology.

Elective Material

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Suggested Program Emphasis (in %) for Required Portion of Mathematics 24

Unit	Suggested Program Emphasis (in %)
Work – Income	8
Work – Income Tax	8
Banking – Personal Banking	8
Banking – Consumer Credits	8
Transportation –Purchasing a Vehicle	8
Transportation–Operating a Vehicle	8
Transportation – Travelling	8
Accommodation – Renting and Purchasing a Home	8
Accommodation – Mortgages and Household Insurance	8
Accommodation – Building, Decorating and Furnishing	8
Cost of Independence – Budgets, and Getting Value for Money Spent	8
Cost of Independence – Life and Health Insurance	8
Elective	20

ATTITUDES

1. *Students will be expected to* demonstrate an attitude associated with mathematical literacy. In particular, students will be expected to:

    1.1 be confident in their mathematical knowledge and in their ability to acquire new knowledge
    1.2 demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems
    1.3 develop intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge
    1.4 exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically
    1.5 be critical and constructive in approaching new ideas and new processes
    1.6 be aware of the importance of communication skills in mathematics
    1.7 appreciate the usefulness of computational competence, mathematical processes and problem-solving skills which are used in the decision-making and modeling processes in our society
    1.8 appreciate the contributions of mathematics to our culture and civilization.

PROBLEM SOLVING

1. *Students will be expected to* demonstrate an understanding of the variety of procedures that can be used to understand problems. In particular, students will be expected to:

    1.1 read the problem thoroughly
    1.2 identify and clarify key components
    1.3 restate the problem, using familiar terms
    1.4 evaluate the given information as to sufficiency and relevancy
    1.5 interpret pictures, charts and graphs
    1.6 determine hidden assumptions
    1.7 ask relevant questions
    1.8 identify given, needed and wanted information
    1.9 diagram or model the problem situation
    1.10 use suitable notation
    1.11 determine valid inferences
    1.12 simulate a problem situation
    1.13 formulate situations into identifiable problems.

2. *Students will be expected to* develop a variety of strategies for use in the solution of mathematical problems. In particular, students will be expected to:

    2.1 conduct an investigation
    2.2 use estimation and approximation
    2.3 develop equations or use formulae
    2.4 use flow charts
    2.5 make lists and charts
    2.6 look for patterns
    2.7 work backward
    2.8 break the problem into smaller parts
    2.9 look for a simpler or related problem
    2.10 make diagrams or models

2.11 use manipulatives
2.12 choose and sequence a series of mathematical operations
2.13 sketch the graph of a problem situation
2.14 establish procedures to gather and organize data
2.15 apply empirical or inductive processes
2.16 use geometric construction and measurement techniques
2.17 make and test a conjecture.

3. *Students will be expected to* develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, students will be expected to:

3.1 apply selected strategies
3.2 present ideas clearly
3.3 document the solution process
3.4 use appropriate group behaviours
3.5 use calculators and computers
3.6 evaluate problem-solving strategies for effectiveness
3.7 alter or abandon non-productive strategies
3.8 search for additional information
3.9 ask questions
3.10 be open to inspirations, intuitions and "bright ideas".

4. *Students will be expected to* employ a variety of skills to help them look back over the solution of a problem. In particular, students will be expected to:

4.1 determine the reasonableness of an answer
4.2 explain the solution in oral or written form
4.3 consider the possibility of additional solutions
4.4 search for other strategies and processes of solution
4.5 create and solve similar problems
4.6 note the characteristics that will be identifiable in similar problems
4.7 make a generalization
4.8 examine the assumptions made and simplifications and modifications used for accuracy, effectiveness and efficiency.

NUMERATION

Numeration concepts and calculation skills are basic in order to address successfully any mathematical situation. Since it is the intent to integrate the development of these concepts and skills into the other expectations of this course, it is not expected that numeration will be taught as a topic on its own.

1. *Students will be expected to* demonstrate an understanding that computation requires skills in addition, subtraction, multiplication and division with whole numbers, integers and rational numbers and can be done mentally, with paper and pencil, and with electronic calculators, as appropriate to a particular situation.

Mental Computation

Students will be expected to:

1.1 add and subtract whole numbers, integers and rational numbers (limit: add three single-digit integers; subtract two single-digit integers; add and subtract two multiples of 5 and 10 up to 100; add and subtract fractions with common denominators; add and subtract tenths and hundredths written in decimal form)
1.2 multiply whole numbers, integers and rational numbers (limit: single-digit numbers, multiples of 5 and 10 multiplied by a single-digit multiplier; double any number up to 50; multiply whole numbers, integers and decimals by powers of 10)
1.3 square integers up to 10 and calculate square roots of integral perfect squares up to 100
1.4 divide whole numbers, integers and rational numbers (limit: divide integers to 100 which are perfect multiples of single-digit divisors; halve even integers to 50; divide whole numbers, integers and decimals by powers of 10)
1.5 determine whether a whole number or integer is divisible by 2, 3, 4, 5 or 10
1.6 transform fractions to percents (limit: denominators of 2, 3, 4, 5, 10, 100) and whole number percents
1.7 calculate the percent of given whole number quantities (limit: 1%, 10%, 25%, 50%, 100%, 200%)
1.8 recognize "families" of fractions (tenths, twelfths, sixteenths) and transform fractions within families to common denominators.

Paper and Pencil Computation

Students will be expected to:

1.9 add and subtract whole numbers, integers and rational numbers (limit: add three 2-digit integers; subtract 3-digit integers; add and subtract two fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 16; add three 2-digit decimals; subtract 2-digit decimals)
1.10 multiply whole numbers, integers and rational numbers (limit: 2-digit integers, 2-digit decimals, fractions with single-digit denominators and denominators that are multiples of 10)
1.11 divide whole numbers, integers and rational numbers (limit: 3-digit integers which are perfect multiples of single-digit divisors; halve even numbers up to 500)
1.12 calculate the percent of given quantities (limit: 5%, 15%, 25% and multiples of 10%)
1.13 calculate the value of a power (limit: whole number base < 10 and exponent of 2 or 3)
1.14 transform mixed numbers to decimals and percents (limit: denominators of 2, 3, 4, 5, 6, 8, 10).

Calculation with Calculators

Students will be expected to:

1.15 calculate the percent of given quantities
1.16 calculate the value of a power (limit: integral base and whole number exponent)
1.17 calculate the square root of any positive rational number
1.18 transform any rational number to decimal and percent form.

2. *Students will be expected to* demonstrate an understanding that rounding, estimation and approximation are fundamental to facility in calculation.

Students will be expected to:

2.1 round given values to a specified level of precision
2.2 round solutions to an appropriate level of precision
2.3 use estimation techniques to predict solutions to calculations.

3. *Students will be expected to* demonstrate an understanding that checking an answer for reasonableness and accuracy is a fundamental step in the calculation process and that the level of accuracy required is dependent upon the particular situation.

Students will be expected to:

3.1 use estimation techniques to check a calculation for reasonableness
3.2 determine if the accuracy of a calculated answer is adequate for a given situation.

GEOMETRY AND MEASUREMENT

Understanding of geometric concepts and facility with measurement skills are basic to many mathematical situations faced in life. Since it is the intent to integrate the development of these concepts and skills into the other expectations of this course, it is not expected that geometry and measurement will be taught as topics on their own.

1. *Students will be expected to* demonstrate an understanding that geometric shapes and solids have mathematical attributes.

Students will be expected to:

1.1 make scale drawings and models of geometric shapes and solids
1.2 use the geometric properties of angles and lines to determine the properties and measurements of shapes and solids.

2. *Students will be expected to* demonstrate an understanding that geometric shapes and solids can be measured directly and indirectly using various techniques, instruments and units, and that the accuracy of any measurement is dependent upon the instrument being used, the individual doing the measurement and the need for precision.

Students will be expected to:

2.1 measure length, angle, area, volume, capacity and mass using appropriate instruments
2.2 determine approximate measurements from scale drawings or maps
2.3 determine the accuracy of a measurement based on the need and the instruments used
2.4 determine the proper precision for an approximated or ascertained measurement
2.5 use formulas for perimeter, area and volume to measure indirectly regular geometric shapes and solids
2.6 use procedures based on known formulas to measure indirectly irregular geometric shapes and solids.

WORK - INCOME
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that pay received for work done can be calculated and that the deductions from gross pay can be determined.	Students will be expected to calculate gross wages based on a regular hourly rate and overtime hourly rate as well as other pay methods, including weekly and monthly salaries, piecework, bonuses, commissions and combinations of methods.	
	Students will be expected to calculate net weekly, biweekly or monthly wages, considering deductions such as unemployment insurance, retirement pensions, Canada Pension Plan, medical insurance, disability insurance and income tax.	Students will be expected to compare the financial advantages between jobs in which the worker gets no benefits other than wages and those in which benefits are provided.
	Students will be expected to calculate gross and net annual earnings, given various pay methods.	
	Students will be expected to calculate weekly, biweekly and monthly earnings, given annual earnings.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11

Geometry and Measurement	2.3 (concept of time)

CONCEPT 1

Students will be expected to demonstrate an understanding that pay received for work done can be calculated and that the deductions from gross pay can be determined.

Students will be expected to:

1.1 calculate gross wages based on a regular hourly rate and overtime hourly rate as well as other pay methods, including weekly and monthly salaries, piecework, bonuses, commissions and combinations of methods

1.2 calculate net weekly, biweekly or monthly wages, considering deductions such as unemployment insurance, retirement pensions, Canada Pension Plan, medical insurance, disability insurance and income tax

  1.2.1 compare the financial advantages between jobs in which the worker gets no benefits other than wages and those in which benefits are provided

1.3 calculate gross and net annual earnings, given various pay methods

1.4 calculate weekly, biweekly and monthly earnings, given annual earnings

COMMENTS

Some questions that can be used when discussing income:

- What type of person would probably do well in each of the following methods of earning income? (provide reasons)
  a. straight commission and b) tips.

- Job A earns on a straight commission of 8% and Job B earns a salary of $700 plus a commission of 5%. Explain why some people will choose Job B even though it earns a lower commission rate.
  (Thanks to J. Frew, St. Francis HS)

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Using local newspapers, have students survey, in groups of 2, "Help Wanted" and "Careers" to determine types of jobs available, pay (if listed), and benefits (if listed). Compare this to information received after surveying students as to the types of jobs, who hires students, what do they pay, what do they expect, what do you learn, and chances for advancement. This can lead to a discussion of the adequacy of student "after school" jobs as potential full time position. Once students have been through this discussion, have each student identify what they would like to do when they have finished school and then predict the annual salary.
(Thanks to B. Wright, Bellerose CHS)

Activity: Have students investigate a variety of jobs in the local community, or in the classified and career sections of newspapers. Determine weekly, monthly, and annual salaries, and, where possible, estimate deductions.

See "Moving Out," Instructor's Guide, pp. 10-11 for Income related terminology and deduction lists. "Moving Out" is a support resource for CALM and is available through LRDC.

Project Activity:

1. Break into groups of 3 or 4.
2. Check through the Career Trends and find a "fantasy" career. You may choose any career so long as it is found in the book.
   (Note: Career Trends is a student reference for CALM 20 and can be obtained through LRDC.)
3. You should list the name of the career, a description of it, the educational and personal requirements needed to fulfil the career goals, the training and the starting salary (it might be a range).

Project Activity: Assignment 1: You are to assume that you have been involved in your career for five years. At the end of each year you have earned, through your hard work and innovative ideas, a raise of 10%. You must calculate your present salary based on the starting salaries given in the Career Trends.

1. What is your starting salary? (If a range is given use the maximum.)
2. What is 10% of your starting salary? (Correct to the nearest cent.)
3. What is your salary at the end of your first year?
4. What is 10% of your previous year's salary? (Correct to the nearest cent.)
5. What is your salary at the end of your second year?
6. Continue in this manner until you have calculated your salary at the end of your fifth year.
7. You have now determined your present yearly salary.
   a. Calculate your monthly salary.
   b. Calculate your weekly salary from your yearly salary.
   c. From your weekly salary calculate your hourly rate. Assume 40h per week.
8. Decide on a name for the company that employs you or the company you own.

Project Activity: Assignment 2: It is now time to look at your future more realistically. If you consider your background (previous job experience), your educational experience (courses you have taken, are taking or will be taking), your future educational opportunities (trade school, SAIT, apprenticeship) and the career goals you presently have, try to find a career in the Career Trends that interests you and is suited to you.

Again list the name of the career, a description of it, educational and personal requirements, the training and the starting salary.

You are to assume that you are starting your job today. You will be paid the lowest starting salary that is listed in the Career Trends. You will receive periodic raises as you earn them.

1. What is your starting salary? (If a range is given use the minimum).
2. Using your starting yearly salary
   a. Calculate your monthly salary.
   b. Calculate your weekly salary from your yearly salary.
   c. From your weekly salary calculate your hour rate. Assume 40h per week.
3. Decide on a name for the company that employs you or the company you own.

Project Activity: Assignment 3: You have been working at your job for a week. Your hourly wage is $12.48 h and you receive time and a half for overtime (more than 8h). Below are the days and hours you worked last week.

Monday – 8h
Tuesday – 10h
Wednesday – 8h
Thursday – 14h
Friday – 6h

Calculate your gross pay.

Along with the normal deductions of CPP, UIC, and income tax, you pay $22.50 per week union dues, $9.00 a week for Alberta Health Care, $12.00 a week for long term disability insurance and $37.50 a week for a private pension plan. Calculate your net pay for the week.

Net claim: $3,550
(Thanks to A. Cooper, Central Memorial H.S.)

Activity: Ask students to bring pay slips from their jobs. They can then analyze and compare benefits, wages and terms of employment.

ELECTIVE SUGGESTIONS

Problem: Assuming you made $40.000 last year, would you rather receive a 15% raise this year, followed by no raise next year, or would you prefer a 7.5% raise each year?

Answer: First is the best

First year: 7.5% raise:	$40.000 (1.075) =	$43.000
Second year: 7.5% raise	$43.000 (1.075) =	$46.225
Total:		$89.225
First year: 15% raise:	$40.000 (1.15) =	$46.000
Second year: 0% raise:	$46.000 (1.00) =	$46.000
Total:		$92.000

A difference of $2,775.00

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RESOURCE CORRELATION

CM:	pp. 18–29, 32–43, 46–59, 79–82, 244–245 (extension), 249 (extension), 318–319 (extension)
CCM:	pp. 133–152, 264–265, 350–352

WORK – INCOME TAX
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that income tax will be paid by all workers and that the amount paid will depend upon a number of factors.	Students will be expected to collect and interpret records of employment income. Students will be expected to determine and calculate allowable deductions from income. Students will be expected to complete and appropriate Revenue Canada taxation form.	Students will be expected to determine the effect of discretionary deductions on total tax paid.

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.7	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14

Geometry and Measurement	

CONCEPT 1

Students will be expected to demonstrate an understanding that income tax will be paid by all workers and that the amount paid will depend upon a number of factors..

Students will be expected to:

1.1 collect and interpret records of employment income
1.2 determine and calculate allowable deductions from income
    1.2.1 determine the effect of discretionary deductions on total tax paid
1.3 complete an appropriate Revenue Canada Taxation form

COMMENTS

You may wish to consider teaching this unit in the second semester when students can prepare their own income tax return.

Note: "Discretionary deductions" in 1.2.1 refer to RRSP's, charitable or political donations, etc.

PROCESS/PROBLEM-SOLVING CONTEXT

See Teaching Taxes, available from Revenue Canada. This is a SUPPORT Career and Life Management 20 resource, consisting of Student Workbook and companion Teacher's Guide, plus a background information booklet, "Taxopedia." The teacher and student resources are updated annually. These materials are not available from LRDC. They may be ordered free of charge, and in class-size quantities from Revenue Canada, Taxation, Public Affairs.

Calgary:
220 – 4 Avenue, S.E.
Calgary, AB
T2G 4X3
231 – 4249

Edmonton:
9700 Jasper Avenue
Edmonton, AB
T5J 4C8
495 – 4770

When ordering, indicate you would like the Alberta form.

Activity: Teaching Taxes provides specific questions about the taxation form, and typical employment and income scenarios. Have students complete some of these problems. Then have students investigate local jobs in which they are typically employed. Determine income, deductions, and tax credits, and income tax.

Activity: Discuss with students the types of employment and life situations people are often engaged in shortly after they complete school. Create scenarios for which income, deductions, tax credits, and income tax can be calculated.

Teachers are encouraged to consult with Career and Life Management teachers to coordinate programs.

TECHNOLOGY INTEGRATION

Use a spreadsheet to investigate various income levels, compulsory and discretionary deductions, and the resulting income tax. Appleworks and Microsoft Works are Alberta Education Basic Resources which provide spreadsheet opportunities.

There is a taxation software package available through:

Cantax Corporation Ltd.
68 Pumpmeadow Crescent, S.W.
Calgary, Alberta
T2V 5C6
Phone: 255-8444
FAX: 253-6281

You may find this useful to teach this topic.

RESOURCE CORRELATION

CM: pp. 50–53, 281–286
CCM: pp. 377–402

BANKING - PERSONAL BANKING
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to be able to demonstrate an understanding that savings and chequing accounts with banks and other financial institutions are services that consumers can use to help manage their money.	Students will be expected to identify and compare types and purposes of accounts. Students will be expected to deposit and withdraw funds.	
	Students will be expected to verify an account statement against records of deposit and withdrawal and reconcile chequing account statements.	Students will be expected to compare the costs of various banking services and determine the advantages and disadvantages of maintaining particular types of accounts.
	Students will be expected to calculate simple interest for periods involving days, months and years.	
	Students will be expected to calculate compound interest.	Students will be expected to use a calculator to calculate compound interest over a maximum of four compound periods. Students will be expected to use tables or a computer to calculate compound interest for more than four compound periods. Students will be expected to compare simple and compound investment earnings amongst various forms of savings plans including types of savings accounts, guaranteed investment certificates and other bank-centred savings plans.

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11, 1.12, 1.14, 1.15

Geometry and Measurement	2.3 (concept of time)

CONCEPT 1

Students will be expected to demonstrate an understanding that savings and chequing accounts with banks and other financial institutions are services that consumers can use to help manage their money.

Students will be expected to:

1.1 identify and compare types and purposes of accounts
1.2 deposit and withdraw funds
1.3 verify an account statement against records of deposit and withdrawal and reconcile chequing account statements
    1.3.1 compare the costs of various banking services and determine the advantages and disadvantages of maintaining particular types of accounts

COMMENTS

Note: Automated Tellers should be included in this discussion as a way to withdraw and deposit money as well as perform other banking services.

An example of other questions that students may want to discuss are:

- What does it mean for a cheque to "bounce"?
- How does the bank "thank you" for doing business with them?

(Thanks to A. Cooper, Central Memorial H.S.)

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students work in groups to survey the types of financial institutions available locally (banks, credit unions, trust companies, treasury branches, etc.). This information may then be organized on large display charts so comparisons can be made of services offered, service charges, and types of accounts. Use this information to create scenarios of banking with these various institutions.

See Moving Out, magazine, p. 24, and Instructor's Guide, p.27.

See also More than Money, from the Canadian Bankers' Association. It is a SUPPORT resource in the Career and Life Management 20 course, available through the LRDC.

Activity: Obtain actual cheque books and other forms from a local bank or have students design their own cheques to make their own cheque book. They can then use these on the following exercise (only a few transactions are provided as an example)

For each deposit you must fill out a deposit slip. For each purchase you must pay by cheque. All deposits and cheques must be entered in your cheque register.

1.	Sept. 25	deposit $1247.76
2.	Sept. 28	purchased groceries at Co-op for $97.53
3.	Oct. 2	mortgage payment to Bank of Montreal of 845.00
4.	Oct. 10	transferred from savings account $300.00
5.	Oct. 12	car payment to GMAC of $250.00
6.	Oct. 12	bought a CD at A&A Records for $21.95

7. Oct. 14 bought a pair of jeans at Work Warehouse for $39.95
8. Oct. 17 deposit $1,353.81
9. Oct. 20 purchased groceries at Co-op for $11.23
10. Oct. 20 purchased bakery items at Glamorgan Bakery for $32.95

Following this assignment, students could reconcile a bank statement, using their cheque register as a basis
(Thanks to A. Cooper, Central Memorial H.S.)

ELECTIVE SUGGESTIONS

Invite a representative of a financial institution to speak to the class about types of services, and verifying account statements and reconciling chequing account statements or arrange for the class to visit a local financial institution. Before either having a guest speaker or going on a visit have the class prepare a set of questions they want answered. To evaluate these type of assignments you can collect the answers obtained.

RESOURCE CORRELATION

CM.	pp. 84–97, 100–107, 277 (extension)
CCM:	pp. 201–213

CONCEPT 1 (continued)

Students will be expected to demonstrate an understanding that savings and chequing accounts with banks and other financial institutions are services that consumers can use to help manage their money.

Students will be expected to:

1.4 calculate simple interest for periods involving days, months and years

1.5 calculate compound interest

- 1.5.1 use a calculator to calculate compound interest over a maximum of four compound periods
- 1.5.2 use tables or a computer to calculate compound interest for more than four compound periods
- 1.5.3 compare simple and compound investment earnings amongst various forms of savings plans including types of savings accounts, guaranteed investment certificates and other bank–centred savings plans

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students examine the difference in the amount of money earned if:

1. $5,000 was deposited at 9%, 10%, 11%, 12%, and 13% over 1, 5, 10, 15, and 20 years vs
2. $1,000 deposited at 9%, 10%, 11%, 12%, and 13% every year for 20 years. Some questions that you may wish to explore:
   - How much does interest rate affect a deposit?
   - How much does time affect a deposit?
   - How can a regulated savings plan grow if "watered regularly"?

(Thanks to B. Wright, Bellerose C.H.S.)

Activity: Complete the activities on compound interest found on the next page.

Problem: Carter had a savings account on which the interest was calculated on the minimum monthly balance. The current rate was 8.25%. His balance for the month of May remained at $980, until May 27, when he withdrew $950. Calculate the amount of interest he lost by withdrawing before month end.

Answer: $6.74 – $0.21 = $6.53

See Moving Out, p. 24, for a discussion on savings.

Activity: Have students collect information about different current interest rates offered for different types of saving programs and different rates at various institutes. You could have students make a collage of the information collected.

TECHNOLOGY INTEGRATION

See CM: p.117, "Computer Keystrokes."

RESOURCE CORRELATION

CM:	pp. 108–115, 304–317
CCM:	pp. 214–220, 426–431

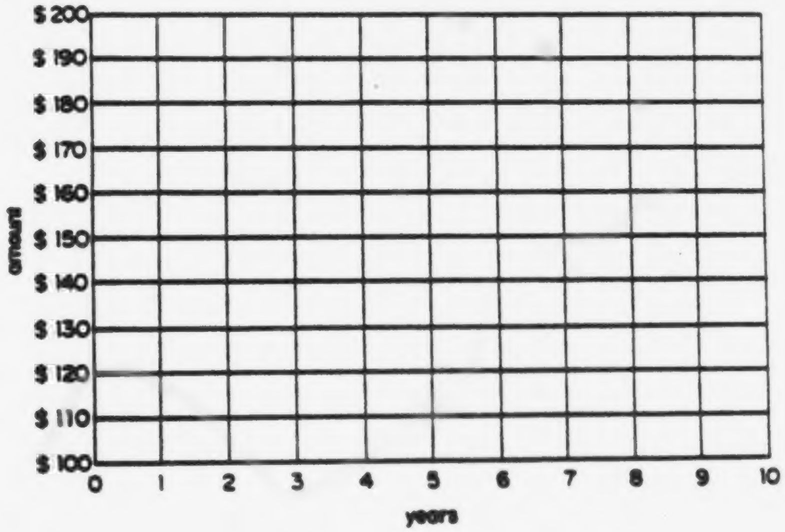
COMPOUNDING

1. Determine what happens to $100 compounded semiannually at 10% over four years. Round down to the nearest cent.

Principal	Semiannual Rate	Interest	Amount		
$100.00	½(10%)	$5.00	$105.00		$1.05^1 \times 100$
$105.00	0.05	$5.25	$110.25	end of year 1	$1.05^2 \times 100$
$110.25	0.05	$5.51(25)	$115.76(25)		$1.05^3 \times 100$
	0.05			end of year 2	$1.05^4 \times 100$
	0.05				
	0.05			end of year 3	
	0.05				
	0.05			end of year 4	

Describe two methods by which you can determine the amount of money available at the end of five years.

2. Using ●'s, graph what happens to $100 invested at 10% simple interest over a period of 10 years. Using x's, graph what happens to %100 compounded semiannually at 10% interest per year over a period of 10 years. Use your answers to exercise 1 above. Discuss the results.



Reprinted from "Compound Interest a la Simple Calculator," in Activities from the Mathematics Teacher copyright 1981 by the National Council of Teachers of Mathematics.

BANKING – CONSUMER CREDIT
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that borrowing money is a consumer activity in which the use of someone else's money is bought and that the price is interest paid on the principle borrowed.	Students will be expected to identify types and characteristics of loans such as consumer loans, car loans and consolidation loans. Students will be expected to calculate the periodic payments for loans using tables. Students will be expected to calculate total amount to be repaid, including interest, given various types of loans and loan characteristics.	Students will be expected to compare actual costs of borrowing between different companies and different types of loans.
Students will be expected to demonstrate an understanding that credit cards provide consumers with instant loans.	Students will be expected to read, interpret and calculate the information on credit card sales drafts and monthly statements. Students will be expected to calculate the interest charge due on a credit card balance. Students will be expected to calculate the service charges associated with using credit cards, including membership charges, yearly fees and transaction fees. Students will be expected to calculate the actual time required and total cost to pay off a credit card balance by paying only the minimum payment.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11, 1.12, 1.14, 1.15

Geometry and Measurement	2.3 (concept of time)

CONCEPT 1

Students will be expected to demonstrate an understanding that borrowing money is a consumer activity in which the use of someone else's money is bought and that the price is interest paid on the principal borrowed.

Students will be expected to:

1.1 identify types and characteristics of loans such as consumer loans, car loans and consolidation loans
  1.1.1 compare actual costs of borrowing between different companies and different types of loans
1.2 calculate the periodic payments for loans using tables
1.3 calculate the total amount to be repaid, including interest, given various types of loans and loan characteristics

COMMENTS

You may wish to include a discussion of the advantages and disadvantages of buying on credit.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students examine the differences in the cost of credit using a bank versus using a "dealer option plan". For example, Bob wants to buy a used boat for $2,289. He can borrow the money from his bank at an annual rate of 15%. He would pay his bank $79.36 per month for 36 months. The dealer offers Bob an annual rate of 21% which means payments to the dealer for $149.33 per month for 17 months. Determine the cost of credit for both the bank and the dealer. Explain how an installment plan with a lower interest rate could actually cost more than one with a higher interest rate.
(Thanks to J.Frew, St. Francis H.S.)

Problem: Ingrid wanted to buy a used car for $2000. She shopped around to borrow the $2000 and was offered the following conditions:

Source	Annual Interest	Number of Payments
1. Financing by the seller	24.00%	36
2. Financing by a lending institution	24.00%	24
3. Financing by a 2nd institution	15.50%	36
4. Financing by a 3rd institution	15.50%	30

5. After making a down payment of $650, she returned to option 4 above. In each case, use a table such as the one below (and CM p. 423) to compare and determine the (a) size of payments, (b) total to pay, and (c) total cost of credit.

Option	Size of Payments	Total of Pay	Total Cost of Credit
1			
2			
3			
4			
5			

Problem: A man borrowed $3 500 and a year later paid back the loan plus interest with a cheque for $4 200. Find the annual rate of interest, in percent paid for the loan.

Answer: 20%

Reprinted from Problem Solving Challenge for Mathematics.

Activity: Since it is important that contexts be meaningful to students, engage in brainstorming a list of borrowing situations in which students or young adults may find themselves. Have students work in groups to choose situations from the list to investigate loan options and the cost of borrowing money in each case. Encourage students to make realistic appraisals of the feasibility of making the purchases and loans given current and anticipated income and expense levels.

Problem: Jill co-signed a $900 loan for her friend when the latter bought a stereo system. The details of the loan were 14.25%/a interest, with 24 monthly payments. After paying for the loan for 8 months, the friend became seriously ill and could not longer make payments. If Jill is to pay off the balance of the loan, how much is still owing of the total loan cost?

Answer: Total loan cost: $1039.59

Balance: $^{2}/_{3} \text{ x } \$1039.59 = \$693.06$
(Monthly payments: $43.32)

See Moving Out, pp.19–23 and pp.25–29 for background discussion and see Moving Out, Instructor's Guide, pp.36–45 for further information.

The following Tipsheets available free of charge from Albert Consumer and Corporate Affairs may also be useful:

- "Consigning Credit Contracts"
- "Credit Worthiness and Credit Reporting Agencies"
- "Responsible Credit Use"

See also the Consumer Talk information sheet "Credit" available from Consumer and Corporate Affairs for further background information and examples for students.

See Appendix F for a list of Alberta Consumer and Corporate Affairs provincial offices, and a general mailing address.

RESOURCE CORRELATION

CM: pp. 140–153
CCM: pp. 226–227, 234–242

CONCEPT 2

Students will be expected to demonstrate an understanding that credit cards provide consumers with instant loans.

Students will be expected to:

2.1 read, interpret and calculate the information on credit card sales drafts and monthly statements
2.2. calculate the interest charge due on a credit card balance
2.3 calculate the service charges associated with using credit cards, including membership charges, yearly fees and transaction fees
2.4 calculate the actual time required and total cost to pay off a credit card balance by paying only the minimum payment

COMMENTS

Specific to expectation 2.4, it is expected that students develop an awareness of the length of time it would take to pay off a credit card balance if only the minimum payment was sent in every month. This is intended to provide students with an understanding of the fact that interest is calculated on any unpaid balance and that it essentially costs to borrow the money. For example: Calculate how long it would take to pay off a credit card balance of $150 if the minimum payment is $25 per month and interest is calculated on any unpaid balance at a rate of 21%.

Students should be involved in finding out the responsibility of credit card companies with respect to the amount of debt that their card holders can manage. For example, do credit card companies have any responsibility? If so, what are ways that they do this?

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students working in groups, choose an article such as a mountain bike to purchase and then investigate the cost of paying for the item using the major credit cards, and, where possible the financing provided by the store from which the item was purchased. Consider various scenarios: paying for the item in a single billing period, paying it off in a fixed period of time (3 months, 6 months, 1 year, for example), or by paying the minimum balance due only.

Problem: Suppose that the balance owing on a major credit card account remains approximately steady at $1500 for a year. That is, monthly payments are approximately equal to new purchases and added interest charges. Calculate the approximate amount of money paid in interest charges for the year, if the interest rate is 21.75%/a.

Answer: Monthly interest charge: $\$1500 \times {}^{.2175}/_{12} = \$27.19$ (approx). Charges for the year: $326.28

See Moving Out pp.19–23 and pp.25–27 for background discussion and see Moving Out, Instructor's Guide, pp.36–45 for further information.

TECHNOLOGY INTEGRATION	See CCM p. 242, "Computer Application: Charge Accounts Program."

RESOURCE CORRELATION

CM:	pp. 122–135
CCM:	pp. 228–233

TRANSPORTATION - PURCHASING A VEHICLE
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding of the costs associated with purchasing a motor vehicle.	Students will be expected to calculate the fair price of a vehicle given age, condition, mileage and options. Students will be expected to determine the monthly payments for installment purchases.	Students will be expected to determine the total costs expected (such as down payment, repairs, insurance, etc.) upon purchase of a vehicle. Students will be expected to compare the costs of various payment options which may be available for the purchase or lease of a vehicle.

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11, 1.11, 1.12, 1.14, 1.15

Geometry and Measurement	2.3, 2.4

CONCEPT 1

Students will be expected to demonstrate an understanding of the costs associated with purchasing a motor vehicle

Students will be expected to:

1.1 calculate the fair price of a vehicle given age, condition, mileage and options.
  1.1.1 determine the total costs expected (such a down payment, repairs, insurance, etc.) upon purchase of a vehicle
1.2 determine the monthly payments for installment purchases
  1.2.1 compare the costs of various payment options which may be available for the purchase or lease of a vehicle

COMMENTS

You may wish to approach car dealerships to collect old cards from cars they sell. This will provide the "real life" element to the discussion. You could arrange for a local car dealer to talk to the class or arrange to visit a local car dealer.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students identify a vehicle that they would like to purchase. Include any options that they would like. Have the students price out the vehicle, with the options, at a local dealership. Once the information is obtained, have students check the public library for probable dealer costs, and work out a "reasonable" offer. This can also be done with purchasing a Used Vehicle. Some questions can examine when "checking out the library" are:

1. How do you calculate a fair price for a vehicle given age, condition, mileage, options, etc?
2.* Select a specific vehicle and summarize all expected costs of owning and operating that vehicle.
3.* Select two similar specific vehicles and make a decision as to which to purchase. Justify your choice and present all information.
4. For similar vehicles compare the costs, advantages, and disadvantages of
   a. purchasing new
   b. purchasing used
   c. leasing
5. How can one avoid the purchase of a "lemon"?
6.* Select ten new vehicles and compare their fuel economics.
7. How are vehicles depreciated? What types depreciate fastest and why?
8. How is vehicle insurance calculated? How do you get the best price on insurance, but still make sure you have adequate coverage?
9. Once a vehicle is purchased, how long should a person anticipate keeping it before trading it in? List as many factors as possible which would enter into making a wise decision as to trade in time.

*Note: Be sure to select vehicles for which information is readily available.

10. What three 1989 vehicles were rated best and why? What three 1989 vehicles were rated worst and why?
11. Approved question of your own making.

(Thanks to J. Haig, Lethbridge Collegiate Institute and E. Hilton, Lindsay Thurber C.H.S.)

Activity: Have students compare the prices of used cars from a used car lot and those listed in the newspaper. Some questions that may be examined:

- Why the difference, if any?
- Is there any guarantee from either?
- What responsibilities of the vendor are their?
- What would you like to know about the vehicle you wish to purchase?
- Could you determine this information, and if so, from what sources?

(Thanks to B. Wright, Bellerose C.H.S.)

Activity: Consider three vehicle purchase possibilities: a new car, a used car, and a new motorcycle. In the case of the used car, identify specific items which need repair. Have students work in groups to investigate realistic purchase and repair prices, the cost of insurance given students' age and the nature of the vehicle, and various loan options. Consider final displays of the results in the form of charts and graphs.

See Moving Out pp.29–30 and the Instructor's Guide, p.48 for background discussion. See also Consumer and Corporate Affairs Tipsheets, "Used Car Buying," and "Things you should know about car consignments," and the Consumer Talk sheet "Buying a Used Car." The Canadian Automobile Association (AMA in Alberta) also provides the pamphlets "Helpful Hints for New Car Buyers," and "Used Car Buying."

For detailed information on motor vehicles, consult such magazines as the Canadian Automobile Association's Autopinion 89 Annual, Consumer Reports "Annual Auto Issue," Road and Track, Motor Trend, Car and Driver, The Canadian Red Book, The Gold Book of Used Car Prices, Lemon-Aid, and the Fuel Consumption Guide

TECHNOLOGY INTEGRATION

See CM: p.173, "Computer Keystrokes"
See CCM: p.266, "Computer Applications: Auto Financing"

See CCM: p. 290, "Computer Applications: Using a Spreadsheet"

Spreadsheets can be used to demonstrate the effect of varying purchase prices, repair costs, insurance costs, and so on. Appleworks and Microsoft Works are Alberta Education Basic Resources which provide spreadsheet opportunities.

RESOURCE CORRELATION

CM:	pp. 158–171,175–176,
CCM:	pp. 251–263, 266, 292–293

TRANSPORTATION - OPERATING A VEHICLE
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding of the costs associated with operating a motor vehicle.	Students will be expected to calculate the costs associated with owning a vehicle including monthly payments, insurance, gasoline and maintenance costs.	
	Students will be expected to calculate the cost of vehicle insurance given factors such as type and age of vehicle, age, gender and driving record of insured.	
	Students will be expected to calculate the fuel economy of a vehicle as rate of consumption per 100 km.	
	Students will be expected to calculate the cost of gasoline for travelling a given distance.	
	Students will be expected to determine the depreciation of a vehicle over time.	
	Students will be expected to calculate the total costs of owning and operating a vehicle over a period of time.	Students will be expected to examine costs, advantages and disadvantages associated with alternatives to owning a motor vehicle, such as using public transit, taxicabs, riding a bicycle, etc.

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11, 1.11, 1.12, 1.14, 1.15

Geometry and Measurement	2.3, 2.4

CONCEPT 1

Students will be expected to demonstrate an understanding of the costs associated with operating a motor vehicle

Students will be expected to:

1.1 calculate the costs associated with owning a vehicle including monthly payments, insurance, gasoline, and maintenance costs
1.2 calculate the cost of vehicle insurance given factors such as type and age of vehicle, age, gender and driving record of insured
1.3 calculate the fuel economy of a vehicle as rate of consumption per 100 km
1.4 calculate the cost of gasoline for travelling a given distance
1.5 determine the depreciation of a vehicle over time
1.6 calculate the total costs of owning and operating a vehicle over a period of time
    1.6.1 examine the costs, advantages and disadvantages associated with alternatives to owning a motor vehicle, such as using public transit, taxicabs, riding a bicycle, etc.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students work through the following scenario:

1. You are a 20 year old man who owns a Firebird (symbol 13).

   Note: "symbol 13" refers to the table in CCM p. 285.

   You are not married and drive 10 km to work. You quit school before you took driver training. You wish to have 100/300/50 liability insurance coverage, collision insurance with $200 deductible and comprehensive insurance with $100 deductible. What is your total six month premium?
2. Two days later you were married to a girl who passed Math 24 with honours. She suggested that you inform your insurance company which you did. A month later you received a cheque in the mail from your insurance company. How much is the cheque?
3. Last year you drove your car 20,000 km. In the year, your car depreciated $2,350, you paid $1,098 for insurance, $42 for plates, $300 for parking, $620 for gas and $285 for maintenance. Calculate the total cost for the year. Calculate the cost per mile rounded to the nearest cent.
4. Unfortunately you were just hit with a speeding ticket and lost your licence for six months. Now you are taking the bus to work and back. If it costs you $1.25 each way, how much will you pay for bus fare during those six months? (Assume there are 20 working days each month.)
5. You also have an active social life but you must take a taxi for the next six months. You estimate that your monthly taxi charges will amount to $240 per month. How much will you spend on taxi during your six month suspension?
6. At the end of the six months you figure that you travelled about 10,000 km. What was your cost per mile using busses and taxis?

7. Will you continue using busses and taxis or go back to using your car? Give two reasons for your decision.
(Thanks to A. Cooper, Central Memorial H.S.)

See Moving Out, pp.28–31, and the Instructor's Guide, pp.47–59 for information and teaching ideas.

See also Consumer and Corporate Affair's Tipsheet "Auto Repairs and Estimates" and their brochure Rental Cars and Insurance.

The Canadian Automobile Association's (AMA) booklet Car Costs, and the similarly titled pamphlet from the Alberta Motor Association provide a detailed cost analysis of operating a vehicle which may be used as a guide for student problems and activities. Consult you local AMA office, or write to:

Alberta Motor Association
Consumer Information Service
P.O. Box 3740, Station "D"
Edmonton, Alberta T5L 4J5
474–8714
Toll Free: 1–800–222–6578

Consult local insurance agents, and the booklet, Car Insurance Explained, Insurance Bureau of Canada, for information on vehicle insurance.

Activity: Insurance Costs. Have students develop several specific, detailed, "case studies."

- type and age of vehicle
- age, gender and driving record of insured
- level of "deductibles,"
- amount of insurance coverage

With this background data, have students consult local insurance agencies to determine annual insurance costs and methods of payment (annual, semi–annually, monthly and so on) and associated premiums

Problem: The Canadian Automobile Association estimates that fuel consumption is increased by 1 to 2 percent for every 45.3 kg of added weight, by as much as 26% with an air conditioner and in stop–and–go driving in hot weather, and up to 15% with an automatic transmission.

(a) Suppose that a particular model of car with a standard transmission averages 8.4 L/100 km under normal city driving conditions. Calculate the increased rate of fuel consumption in the same model car in a "worst case" scenario, carrying a 90 kg passenger for a week in July during 30° weather, while driving home in city traffic with an automatic transmission.

Answer: Passenger: 4% added fuel consumption
Air conditioner: 26% added fuel consumption
Automatic transmission 15% added fuel consumption
Total rate of increase in fuel consumption: 45%

Fuel consumption: $^{8.4L}/_{100\ km}$ x 1.45 = $^{12.2L}/_{100\ km}$

(b) If the drive home is 10 km, how much extra fuel is consumed over the five day period?

Answer: Distance travelled: 5 x 10 km = 50 km
"Normal" average fuel consumption: 4.2 L
"Worst case" average fuel consumption: 6.1 L

(c) What is the extra cost if the driver uses regular unleaded gasoline at $45.8^{¢}/_{L}$?

Answer: Extra cost: $1.9\ L \times {}^{\$0.458}/_{L} = \$0.87$

The extra cost appears to be minimal. Stress that the distance travelled and the time period considered were short. Emphasize the substantial rate of increase in fuel consumption.

Problem: Marie drives a car which averages ${}^{9.2L}/_{100\ km}$ in city driving. She estimates she drives 250 km in a week in the city. After examining a city map, she decided that 40% of her weekly travelling could be done on a bicycle.

(a) Calculate her approximate fuel savings if she put her plan into effect for four weeks. Assume the gasoline she used cost $44.3^{¢}/_{L}$.

Answer: $16.30

(b) If she were to keep to this pattern from May through September, what would her fuel savings be?

Answer: $89.70 (22 weeks)

(c) What other expenses would decrease? increase?

Answer: discuss

For a list of fuel consumption of many vehicles, see the Fuel Consumption Guide, available free from Transport Canada, Public Affairs Branch, or through agencies such as the Alberta Motor Association.

TECHNOLOGY INTEGRATION

See CM: pp.344-345, "A Car Expenses Program"
See CCM: .p.290, "Computer Applications: Using a Spreadsheet."

RESOURCE CORRELATION

CM: pp. 164-171, 175-178, 228-229
CCM: pp. 272-290

TRANSPORTATION – TRAVELLING
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding of the costs associated with travelling.	Students will be expected to calculate the costs of accommodation and meals based on the location, length of trip, length of stay, type of meal plan and gratuities. Students will be expected to estimate the costs of automobile travel, including fuel, maintenance, accommodation and meals en route and compare the costs to other modes of travel. Students will be expected to estimate the total distances travelled using maps and mileage charts. Students will be expected to estimate miscellaneous costs such as entertainment, excursions, gratuities, health insurance and travel insurance. Students will be expected to calculate the costs associated with travellers cheques and conversions between currencies. Students will be expected to compare the costs of package vacations to the estimated costs of individually arranged vacations.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.18	1.9, 1.10, 1.11, 1.11, 1.12, 1.14, 1.15

Geometry and Measurement	2.3, 2.4

CONCEPT 1

Students will be expected to demonstrate an understanding of the costs associated with travelling.

Students will be expected to:

1.1 calculate the costs of accommodation and meals based on the location, length of trip, length of stay, type of meal plan and gratuities
1.2 estimate the costs of automobile travel, including fuel, maintenance, accommodation and meals en route and compare the costs to other modes of travel
1.3 estimate the total distances travelled using maps and mileage charts
1.4 estimate miscellaneous costs such as entertainment, excursions, gratuities, health insurance and travel insurance.
1.5 calculate the costs associated with travellers cheques and conversions between currencies
1.6 compare the costs of package vacations to the estimated costs of individually arranged vacations

COMMENTS

In addition to the items outlined in the expectations, you may wish to discuss sales taxes that are present in many provinces and states.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Plan a two week trip in western Canada. Determine a destination and travel distances by road. Consider forms of travel: personal or rental vehicle, bus, train, plane. (More than one form of travel over two weeks may need to be considered.) Estimate expenses related to different forms of accommodation which enroute, and at the destination. Consult travel agencies, and prepare a table of travel costs for each mode of travel. Consult financial institutions for travellers' cheque costs. The results may be displayed in chart and graph forms.

See Moving Out, pp. 28–29, for a brief discussion of travel expenses and alternatives.

Consult financial institutions and travel agencies for information packages, pamphlets and brochures pertaining to this concept.

Consider having a resource person from the tourism industry to speak to the students.

RESOURCE CORRELATION

CM: pp. 170–171
CCM: pp. 296–306

ACCOMMODATION – RENTING AND PURCHASING
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that renting and purchasing a residence are two options with different responsibilities, advantages and costs.	Students will be expected to investigate the costs of renting given various conditions such as location, type of building, size and amenities. Students will be expected to estimate the monthly costs associated with renting a residence, including rent, parking, insurance and other fixed costs and utilities, and other variable costs. Students will be expected to investigate the purchase price of a residence given various conditions such as type of housing, location, amenities and size. Students will be expected to calculate the costs and fees associated with purchasing a residence. Students will be expected to estimate and calculate the monthly costs associated with home ownership, including mortgage payments, taxes, utilities, insurance and maintenance. Students will be expected to calculate the costs and fees associated with moving into a residence, such as moving, utility hook-ups and security deposit. Students will be expected to estimate and calculate the costs of furnishing and supplying a rental accommodation. Students will be expected to compare the relative costs and advantages between owning and renting property.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14,

Geometry and Measurement	2.3, 2.4

CONCEPT 1

Students will be expected to **demonstrate an understanding that renting and purchasing a residence are two options with different responsibilities, advantages and costs.**

Students will be expected to:

1.1 investigate the costs of renting given various conditions such as location, type of building, size and amenities
1.2 estimate the monthly costs associated with renting a residence, including rent, parking, insurance and other fixed costs and utilities and other variable costs.
1.3 investigate the purchase price of a residence given various conditions such as the type of housing, location, amenities and size
1.4 calculate the costs and fees associated with purchasing a residence
1.5. estimate and calculate the monthly costs associated with home ownership, including mortgage payments, taxes, utilities, insurance and maintenance
1.6 calculate the costs and fees associated with moving into a residence, such as moving, utility hook-ups and security deposit
1.7 estimate and calculate the costs of furnishing and supplying a rental accommodation
1.8 compare the relative costs and advantages between owning and renting property

COMMENTS

Teachers are encouraged to consult with Career and Life Management 20 teachers regarding resource material, teaching activities and strategies and program coordination.

See Moving Out pp.4–13, and the Instructor's Guide, pp.12–20, 23, and 63 for information and teaching ideas.

See also Consumer and Corporate Affairs Tipsheets "Household Moving," "Inspecting a House" (this sheet provides a detailed home checklist suitable for activities involving estimating the cost of repair or improvement), "Buying a House," The Landlord and Tenant Act," "Mobile Homes," "Modular Housing," and "Renting a Mobile Home Site." The Consumer Talk information sheets "Tenants: Moving Out," are also available for students.

Tailor the contextual problems and activities to conditions found in the local community. For example, purchasing a mobile home may be a common option in some locales.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Paying all the utilities is a usual fact of life when owning a home, and sometimes when renting. Investigate the type of utilities that must be paid for in the local community – a power, gas, telephone, water, sewer, other – and the fixed and unit costs associated with each. C contact the appropriate utility companies to determine these costs and rates. Create a table of these costs and rates suitable for use in specific problems.

Activity: Have students consider various rental situations (shared accommodation, single basement suite, etc.) and then use a chart similar to that shown to determine costs. This activity requires students to contact their local utilities companies to determine hook-up charges. At the same time, they should determine unit costs and fixed costs for utilities. This will enable students to determine such things as appliance-use costs, and estimate average utility costs.

MOVING IN COSTS

Item		Your share
Moving Expenses		
First Month's Rent and Security Deposit		
Utility Hook-up Charges	telephone	
	power	
	gas	
	water	
	hook-up deposits	
	cable	
Personal Property Insurance		
Staple Food Items		
First Week's Groceries (or until pay day)		
Kitchen Utensils and Dinnerware		
Linens and Bedding		
Furniture		
Other		
Total Moving in Costs		

Reprinted from Moving Out, Alberta Consumer and Corporate Affairs.

Activity: Use the "Basic Equipment and Furniture for Setting up House" guide. Have students, working in groups, choose the "Living Room" or "Kitchen" settings and investigate the cost (from different sources such as catalogues, magazines, and sales flyers, or by surveying the costs in local stores) of each of the items. Consider total costs for various "have" and "need" combinations.

BASIC EQUIPMENT AND FURNITURE FOR SETTING UP HOUSE

Living Room	Have	Need	$
sofa			
end table(s)			
lamp(s) and shades			
shelving units			
stereo			
radio			
television			
coffee table			
window treatment (curtains, blinds)			
chair			
Kitchen	**Have**	**Need**	**$**
table and chairs			
window treatment			
waste basket			
dish drainer and mat			
dishes (for 4 or 8)			
glassware			
flatware			
dish towels			
pot holders			
clock			
dish cloth or sponges			
garbage bags			
wax paper, plastic wrap, foil			
plastic storage bags			
tablecloth/placements			
drawer organizers			
cleanser			
scouring pads			
dish detergent			
all-purpose liquid cleaner			
bucket and mop			
broom and dustpan			
pots and pans			

Reprinted from Moving Out Instructor's Guide, Alberta Consumer and Corporate Affairs.

Problem: The estimated cost of using a 600W block heater for 10 hours daily is $9.00 per month (30 days), $12.60 if plugged in for 14 hours per day and $52.50 for 24 hours per day (Edmonton Power rates). During the cold month of January, Sarah plugged her car in 10 hours per day for 10 days, 14 hours per day for four days, and 24 hours per day for the remaining time (away on holidays). Calculate the approximate cost of using the block heater for the month (ignore the 31st day)

Answer: Cost = $^{10}/_{30} \times \$9.00 + {^{4}/_{30}} \times \$12.60 +$
$^{16}/_{30} \times \$52.50 = .32.68$

Consult local power and gas utility companies for information regarding appliance operating costs. Contact the local electrical power company or the Department of Energy, Mines and Resources Canada, Communications Distribution:

580 Booth Street
Ottawa, Ontario
K1A 0E4
(613) 995–3065

for Energuide Directories. These list the average monthly electricity consumption of all makes of a given appliance sold in Canada. They are available from a number of appliances, including clothes dryers, freezers, ranges, and refrigerators.

Problem: When Carla and her husband Rick went shopping for a new range, freezer, and clothes dryer, they chose a stove with an energy consumption rating of 63 $^{\text{kwh}}/_{\text{month}}$, a freezer with a rating of 98 $^{\text{kwh}}/_{\text{month}}$, and a clothes dryer with a 74 $^{\text{kwh}}/_{\text{month}}$ rating. Assuming an average utility rate of $\$0.05/_{\text{kwh}}$, calculate the monthly and annual costs of operating these appliances.

Answer: Monthly: $3.15 + $4.90 + $3.70 = $11.75
Annual: $141.00

TECHNOLOGY INTEGRATION

Spreadsheets could be used to create tables of values which may be modified by changing rates, fixed costs, and so on.

RESOURCE CORRELATION

CM: pp. 186–189, 196–197, 202–215, 217–222, 240–241
CCM: pp. 315–323, 344–345, 348–349

ACCOMMODATION – MORTGAGES AND HOUSEHOLD INSURANCE PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that a mortgage is a loan used to purchase a residence.	Students will be expected to define mortgage and distinguish between mortgage types and options. Students will be expected to calculate mortgage payments using amortization tables for various terms and conditions. Students will be expected to calculate the total cost of purchasing real estate over the entire length of the amortization period. Students will be expected to estimate and calculate the regular payment made on a residence if it is comprised of principle, interest and taxes.	Students will be expected to compare the costs of mortgage payments given various terms and payment options. Students will be expected to compare the total costs of several payment plans.
Students will be expected to demonstrate an understanding that household insurance is protection purchased from a company to cover losses incurred to a residence or its contents.	Students will be expected to examine types of household insurance policies and the purposes and advantages of each. Students will be expected to calculate the costs of insurance based on various factors including types of coverage, location, property value, type of construction, availability of fire protection, amount deductible and limits of liability.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14,

Geometry and Measurement	2.3, 2.4

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CONCEPT 1

Students will be expected to demonstrate an understanding that a mortgage is a loan used to purchase a residence.

Students will be expected to:

1.1 define mortgage and distinguish between mortgage types and options
1.2 calculate mortgage payments using amortization tables for various terms and conditions
  1.2.1 compare the costs of mortgage payments given various terms and payment options
1.3 calculate the total cost of purchasing real estate over the entire length of the amortization period
  1.3.1 compare the total costs of several payment plans
1.4 estimate and calculate the regular payment made on a residence if it is comprised of principal, interest and taxes

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students make a list of all things in their bedroom that they would have to replace in the event of a fire (e.g., bed, stereo, dresser, clothes). Do some quick catalogue shopping and determine the replacement cost if no insurance was available.
(Thanks to A. Cooper, Central Memorial H.S.)

Activity: Have students work in groups to investigate the mortgage rate and terms offered by local financial institutions. Use this information to compare costs, given different options.

See the Alberta Consumer and Corporate Affairs Tipsheet "Mortgages." Also consult local financial institutions regarding mortgage information.

TECHNOLOGY INTEGRATION

See CM: p.191, "Computer Keystrokes".

Use a spreadsheet to allow students to explore the costs of various payment plans.

RESOURCE CORRELATION

CM: pp. 180–187
CCM: pp 335–343, 346–347

CONCEPT 2

Students will be expected to demonstrate an understanding that household insurance is protection purchased from a company to cover losses incurred to a residence or its contents.

Students will be expected to:

2.1 examine types of household insurance policies and the purposes and advantages of each

2.2 calculate the costs of insurance based on various factors including types of coverage, location, property value, type of construction, availability of fire protection, amount deductible and limits of liability

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: It is important for students to closely consider all the possessions they might have or would like to have shortly after they moved in, after a year or so, and after several years. Have them consider tenants insurance and its cost by completing a personal property inventory twice, the first time based on a "moving in" estimate of belongings, and a second "one or two years later" estimate. By surveying catalogues, stores, and sales flyers, students can then estimate the present replacement cost of the items. Use the attached Insurance Bureau of Canada form for the inventory.

Students should be realistic in their estimate of possessions. That is they are not expected to indicate that they own everything on the list! Students should skip the "Date Purchased" and "Purchase Price" columns. After completing the list and establishing totals, students should then investigate the cost of tenants insurance by contacting local insurance agencies and obtaining rates for various options.

See Moving Out, p.13 and the Alberta Consumer and Corporate Affairs Tipsheet "A Guide to Insurance Claims: Property and Auto."

The Insurance Bureau of Canada also offers booklets and pamphlets such as Home Insurance Explained, and "Personal Property Inventory," and a Catalogue of Films, Videos and Publications. Also contact local insurance agencies for information.

RESOURCE CORRELATION

CM:	pp. 198–201, 215
CCM:	pp. 320–321, 346–347

ACCOMMODATION – BUILDING, DECORATING AND FURNISHING
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that the approximate cost of building a new house can be determined through estimations based on known measures and costs.	Students will be expected to make a scale drawing of a house and a lot.	
	Students will be expected to use the local building code ratios between the size of the foundation, house and lot to determine the allowable size of the house.	
	Students will be expected to estimate the total cost of building a house based on estimated costs per square metre.	Students will be expected to compare the costs of building a house using various combinations and qualities of building materials.
Students will be expected to demonstrate an understanding that a residence can be represented by a scale drawing, which can then be measured and used for various purposes.	Students will be expected to make a scale drawing of a house, apartment or room to be decorated and furnished.	
	Students will be expected to calculate the surface areas (floors, ceilings, walls) to be decorated.	
	Students will be expected to calculate the costs of floor, window and wall coverings and the total costs.	Students will be expected to perform an investigation to determine the costs of redecorating an entire residence.
	Students will be expected to calculate the costs of furnishings and appliances.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14,

Geometry and Measurement	1.1, 1.2, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

CONCEPT 1

Students will be expected to **demonstrate an understanding that the approximate cost of building a new house can be determined through estimations based on known measures and costs.**

Students will be expected to:

1.1 make a scale drawing of a house and a lot
1.2 use the local building code ratios between the size of the foundation, house and lot to determine the allowable size of the house
1.3 estimate the total cost of building a house based on estimated costs per square metre
  1.3.1 compare the costs of building a house using various combinations and qualities of building materials

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: (1) Have students begin by sketching a floor plan of their proposed house. They should use squared or graph paper.

(2) Have students pass their sketches to others for constructive critique. Students should be prepared to justify their sketches. The sketches should be reviewed for such things as:

- appropriately sized and placed rooms, hallways, staircases, etc. Are these features sufficiently large to accommodate easy movement, necessary furniture and appliances, etc. Are they too large? Are some of these features misplaced?
- missing rooms, hallways, stairways, etc.
- missing, inappropriately sized, or misplaced doors, windows, closets, and so on.

(3) when the sketch has been adequately checked and modified where necessary, the students can choose an appropriate scale and draw the house (and lot) to scale.

Activity: Have students estimate the surface area of the exterior walls of their house. Contact building supply stores in the area for unit costs of various forms of exterior covering: siding (vinyl, aluminum, cedar, particle board, etc.), brick, stucco/plaster-type, etc. Calculate the cost of covering the walls with a single type or a combination of types of these materials.

RESOURCE CORRELATION

CM: pp.
CCM: pp. 358–370

CONCEPT 2

Students will be expected to demonstrate an understanding that a residence can be represented by a scale drawing, which can then be measured and used for various purposes.

Students will be expected to:

2.1 make a scale drawing of a house, apartment or room to be decorated and furnished
2.2 calculate the surface areas (floors, ceilings, walls) to be decorated
2.3 calculate the costs of floor, window and wall coverings and the total costs
    2.3.1 perform an investigation to determine the costs of redecorating an entire residence
2.4 calculate the costs of furnishings and appliances

COMMENTS

This unit will interest many students. It provides a student the opportunity to use their creativity in designing and decorating a house, apartment or room.

PROCESS/PROBLEM-SOLVING CONTEXT

Activity: Have students design their "ideal" house, apartment or bedroom. Students must provide with following information when this "project" is complete:

- scale diagram of the surface areas of the floors, ceilings and walls
- calculations of the surface areas of the floors, ceilings and walls
- the type of product they would use to decorate the floors, ceilings and walls; why they chose that product over an alternative; plus a price comparison (collected either by examining sales flyers or catalogues or by visiting different stores) of the products. For example: if I decide to use wallpaper rather than paint to decorate the wall - I'd indicate why I chose wallpaper and then identify the price comparisons between the wallpaper and paint and then between different brands of wallpaper.
- given the product they chose to use to decorate the floors, ceilings and walls, calculate the total cost of decorating - you may wish to extend this to "Who will do the actual work in the decorating?" This could then include an examination of the time and wages that would be involved in hiring tradespeople to do the work vs the cost of doing this project themselves.
- the type of furnishings and appliances they would use. Once again, a comparison of the products, costs of products, and any labour associated with the products could be included.
- an extension: calculate the amount of material that would be needed to build the house, apartment or bedroom; calculate the total cost of labour and material.

RESOURCE CORRELATION

CM: pp. 224–227, 230–233
CCM: pp 358–370

COST OF INDEPENDENCE - BUDGETS AND GETTING VALUE FOR MONEY SPENT
PROGRAM EMPHASIS - 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that a personal budget is useful in order to maintain records of expenditure and to plan for the future.	Students will be expected to record estimated earnings and expenses. Students will be expected to calculate individual expenses as percents of total expenses. Students will be expected to design and balance a personal budget.	
Students will be expected to demonstrate an understanding that consumers can obtain the best value for their money by the application of some wise spending practices.	Students will be expected to calculate and compare the unit costs of items. Students will be expected to calculate the sale price of an item given the list or regular price and the discount. Students will be expected to calculate the rate of discount given the regular and the sale prices. Students will be expected to calculate the regular price given the sale price and the rate of discount. Students will be expected to calculate the sales tax on purchases.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14,

Geometry and Measurement	2.3

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CONCEPT 1

Students will be expected to demonstrate an understanding that a personal budget is useful in order to maintain records of expenditure and to plan for the future.

Students will be expected to:
1.1 record estimated earnings and expenses
1.2 calculate individual expenses as percents of total expenses
1.3. design and balance a personal budget

PROCESS/PROBLEM-SOLVING CONTEXT

See Moving Out, pp. 14–23, and the Instructor's Guide, pp. 14–31 for information and teaching ideas for preparing a budget.

Activity: Have students work in groups to develop realistic budgets for various single and shared-accommodation situations. Use the attached Budget Planner sheets (from Alberta Consumer and Corporate Affairs). Encourage students to be realistic in their estimates.

This activity will require time; it draw heavily upon the information acquired in previous units, as well as needing new information. It may be advisable to divide the information collecting activities indicated by the planner among the groups of students.

Project: This project is related to budgeting but will ask you to calculate a number of things that relate to previous work.

In our Delorean we have raced into the future to find you, at 28, married to a 28 year old and having two preschoolers (twins). Both of you work and have a combined monthly NET income of $4,350. You live in a comfortable home but carry a mortgage of $94,000 at 12.5% amortized over 25 years. The lady of the house recently bought a car worth $10,200 which you are financing over 36 months at 13%. The man of the house also owns a car. You each take your car to work and each of you travel about 12 km to work. Both cars are rated as symbol 7 (note: "symbol 7" refers to the table in CCM p. 285) and have 100/300/50 liability insurance, $100 deductible collision insurance and $100 deductible comprehensive insurance. Each of you are in the third year of a term life insurance policy that covers each of you for $125,000. From the information just given, you must calculate the monthly payments for the house, car, car insurance and life insurance. You also have other payments, which are listed below, some of them are monthly, some are weekly, some are every two weeks and so on. You must read each one carefully and calculate the monthly charge. List each payment and the amount and then determine how much money you are able to save each month.

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Other Payments

1. Daycare - $504 per month
2. Groceries - $110 per week
3. Retirement Plan - $300 per month
4. Natural Gas Bill - $33 per month
5. Utilities Bill - $72 per month
6. Cable TV - $36 every two months
7. Gas for the car - $50 every two weeks
8. Home Insurance - $200 per year
9. House Cleaner - $45 every two weeks
10. All other expenses - $450 per month

(Thanks to A. Cooper, Central Memorial H.S.)

TECHNOLOGY INTEGRATION

The budgeting process lends itself well to the use of the computer. Specific commercial "budget" software is available. Spreadsheets also provide budget activity opportunities. Appleworks and Microsoft Works are Alberta Education Basic Resources with spreadsheet capabilities.

RESOURCE CORRELATION

CM: pp. 268–275, 155 (extension), 324–334 (extension)
CCM: pp. 178–195

BUDGET PLANNER

MONTHLY TAKE-HOME INCOME

WAGES/SALARY	
WAGES/SALARY	
FAMILY ALLOWANCE/PENSION	
OTHER REGULAR MONTHLY INCOME	
TOTAL MONTHLY TAKE-HOME INCOME	

MONTHLY BUDGET

MONTHLY SAVINGS	EMERGENCY FUND	
	TOTAL MONTHLY SAVINGS	1

MONTHLY LIVING EXPENSES	FOOD (plus other grocery store items)		
	HOUSING	1st mortgage or rent	
		2nd mortgage	
	UTILITIES (total from box "A" on opposite page)		
	HOUSEHOLD INCIDENTALS (dry cleaning, newspapers, etc.)		
	TRANSPORTATION	gasoline	
		bus fare	
		parking	
	PERSONAL ALLOWANCES AND RECREATION (total from box "B" on opposite page)		
	OTHER (child support, alimony, child care, household help, etc.)		
	MONTHLY REQUIREMENT FOR IRREGULAR AND ANNUAL EXPENSES (total from box "C" on opposite page)		
	TOTAL MONTHLY LIVING EXPENSES		2

MONTHLY CREDIT PAYMENTS		
	TOTAL MONTHLY CREDIT PAYMENTS	3

TOTAL MONTHLY BUDGET (add lines 1, 2, and 3)	

Reprinted from Moving Out Instructor's Guide, Alberta Consumer and Corporate Affairs.

BUDGET DETAILS

Transfer the totals from boxes A, B, and C to the appropriate line on the MONTHLY BUDGET chart on the opposite page

A MONTHLY UTILITIES		
POWER		
WATER/SEWAGE/WASTE		
TELEPHONE	local	
	long distance	
NATURAL GAS		
CABLE TV/PAY TV		
TOTAL MONTHLY UTILITIES		

B MONTHLY PERSONAL ALLOWANCES AND RECREATION		
PERSONAL ALLOWANCES (monthly amount for each family member for lunches, grooming, hobbies, tobacco, alcohol, etc.)		
FAMILY RECREATION		
TOTAL MONTHLY PERSONAL ALLOWANCES AND RECREATION		

NOTES

C IRREGULAR AND ANNUAL EXPENSES (All figures on this chart should be annual not monthly amounts.)		
CLOTHING (list annual amount for each family member)		
INSURANCE (if you pay directly)	vehicles	
	life	
	property	
	other	
MEDICATION/MEDICAL FEES		
DENTAL/OPTICAL		
EDUCATION	tuition	
	books & supplies	
TAXES (if you pay directly)	property	
	income	
LICENSES	vehicles	
	other	
MAINTENANCE	vehicles	
	home & garden	
	furnishings	
GIFTS/FESTIVITIES	Christmas	
	other	
TRAVEL/HOLIDAYS	annual holiday	
	other travel	
CONTRIBUTIONS/DONATIONS		
MEMBERSHIPS/SEASON TICKETS		
OTHER (subscriptions, items you plan to purchase next year, etc.)		
TOTAL IRREGULAR AND ANNUAL EXPENSES		
MONTHLY REQUIREMENT FOR IRREGULAR AND ANNUAL EXPENSES (divide total by 12)		

Reprinted from Moving Out Instructor's Guide, Alberta Consumer and Corporate Affairs.

CONCEPT 2

Students will be expected to demonstrate an understanding that consumers can obtain the best value for their money by the application of some wise spending practices.

Students will be expected to:
2.1. calculate and compare the unit costs of items
2.2 calculate the sale price of an item given the list or regular price and the discount
2.3. calculate the rate of discount given the regular and the sale prices
2.4 calculate the regular price given the sale price and the rate of discount
2.5 calculate the sales tax on purchases

PROCESS/PROBLEM-SOLVING CONTEXT

See Moving Out, pp. 32–35, and the Instructor's Guide, pp. 64–67 for background information and teaching idea.

See also Alberta Consumer and Corporate Affairs Tipsheets "Shop Around" and "Mail-Order Buying," and their Consumer Talk information sheets "Shop Around," "Returns," and "Shopping at Home."

Activity: Have students examine sale flyers to determine if the sale price is a true saving. The following procedure is suggested: (1) find a number of items on sale in the flyer. (2) calculate the sale price of the goods according to (a) percent discount (b) amounts deducted. (3) decide if the item on sale is a bargain or not. Some comparison shopping may be necessary here. (4) justify your reasons for saying why the sale item is a bargain or not.

Activity: Have students create their own percentage "sales" problems. If computers with word processing and graphics capabilities are available, have small groups of students create their own "sales flyers." Some intentional errors may be include. These may then be checked by other groups. For further ideas sed "Using Percent Problems to Promote Critical Thinking," Mathematics Teacher, January 1988.

Activity: Have students keep a record of how they spend their money over the period of one week. Through class discussion, develop a standard set of expense categories. At the start of the week, ask students to estimate what percent of their money they will spending each category. At the end of the week, have students calculate the actual percent of their money spent in each classification. A discussion on budgeting could follow.

Problem: Use a calculator to solve this problem. A 37.5% discount was offered on a $750 stereo. However, a 7% GST was added. What was the total amount of money which would have to be spent to purchase the stereo?

Problem: An item is discounted 25% for a sale, another 10% because of the size of the purchase, and 3% for paying cash. These discounts are taken one after the other. The customer paid $195.89. What was the original purchase price?

Answer: $299.18

Problem: Tom bought $30 jacket at a 20% discount and a $20 shirt at a 30% discount. What is the single percentage discount he received on his total purchase?

Answer: 24%

TECHNOLOGY INTEGRATION

Video: Math Wise: (ACCESS) "Comparing 01 – Numerical Comparisons"

ELECTIVE SUGGESTIONS

Problem: The value of a machine declines by 10% for each year of use. If the original cost of the machine was $5000, after a minimum of how many full years will its value be less than 50% of its original value?

Answer: 7 years
$(0.9)^7 < 0.5$.

This problem can be solved by a process of iteration. Suggest students set up an appropriate table.

RESOURCE CORRELATION

CM: pp. 62–77, 236–247
CCM: pp. 42–46, 160–174

COST OF INDEPENDENCE – LIFE AND HEALTH INSURANCE
PROGRAM EMPHASIS – 8%

Concept	Skills	Problem Solving/ Technology
Students will be expected to demonstrate an understanding that life insurance is a way to provide financial protection for dependents in case of death.	Students will be expected to identify, compare and discuss features of types of life insurance such as term, whole–life insurance such as term, whole-life, endowment and group. Students will be expected to calculate premiums for life insurance coverage given factors such as amount of insurance, age, gender, health, employment, life–style and leisure activities. Students will be expected to identify and examine company projections of life insurance values, including dividends, over the life of a policy. Students will be expected to calculate the costs and benefits of health insurance given various types of coverage. Students will be expected to calculate the costs and benefits of disability insurance.	

Integration of Numeration/Geometry and Measurement Concepts and Skills

The numeration/geometry and measurement concepts and skills identified here represent those concepts and skills that students should use, reinforce, or learn within this unit of study. A detailed description of each skill can be found on pages 222 and 223.

	Mental	Estimation	Technology	Paper & Pencil
Numeration	1.1, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8	2.1, 2.2, 2.3, 3.1, 3.2	1.15, 1.18	1.9, 1.10, 1.11, 1.12, 1.14,

Geometry and Measurement	2.3

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CONCEPT 1

Students will be expected to demonstrate an understanding that life insurance is a way to provide financial protection for dependents in case of death.

Students will be expected to:

1.1 identify, compare and discuss features of types of life insurance such as term, whole-life, endowment and group
1.2 calculate premiums for life insurance coverage given factors such as amount of insurance, age, gender, health, employment, life-style and leisure activities
1.3 identify and examine company projections of life insurance values, including dividends, over the life of a policy
1.4 calculate the costs and benefits of health insurance given various types of coverage
1.5 calculate the costs and benefits of disability insurance

COMMENTS

Teachers are encouraged to consult with the Career and Life Management 20 teachers to better coordinate activities and the use of resources (activities, information packages, resource persons, etc.)

PROCESS/PROBLEM-SOLVING CONTEXT

Bring in a representative from the Life and Health Insurance field. Based on the information obtained in this visit, various activities may be set up.

RESOURCE CORRELATION

CM:	pp. 288–303
CCM	pp. 406–420

REVERSE TEXTUAL CORRELATION
GAGE EDUCATIONAL PUBLISHING COMPANY
CONSUMER AND CAREER MATHEMATICS
(Second Canadian Edition)

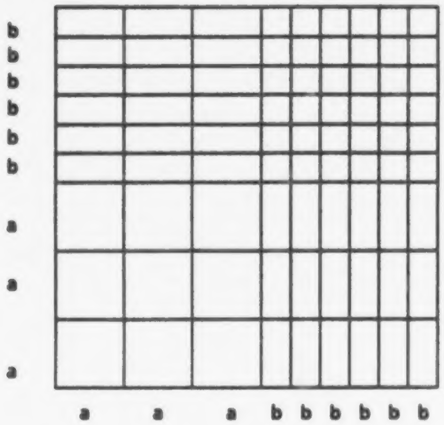
Page	Unit	Learner Expectation Number	TRM Page
2–22	Number Skills Review		
24–29	Solving Linear Equations		
30–41	Ratio, Proportion, and Percent		
42–46	Cost of Independence – Budgeting	2.2, 2.3, 2.4	283
50–51	Units of Measure		
50–59	Geometry and Measurement		
60–70	Geometry and Measurement		
71–73	Cumulative Review and Test		
74–94	Problem Solving		
95–110	Probability		
111–128	Statistics		
129–131	Cumulative Review and Test		
133–143	Work – Income	1	227
144–152	Work – Income	1	227
156–159	Nutrition		
160–174	Cost of Independence – Budgeting	2	283
178–195	Cost of Independence – Budgeting	1	279
201–213	Banking – Personal Banking	1.1, 1.2, 1.3	237
214–220	Banking – Personal Banking	1.4, 1.5	239
226–227	Banking – Consumer Credit	1.1	243
228–233	Banking – Consumer Credit	2	246
234–242	Banking – Consumer Credit	1	243
251–263	Transportation – Purchasing a Vehicle	1	251
264–265	Work – Income	1	227
266	Transportation – Purchasing a Vehicle	1	251
272–290	Transportation – Operating a Vehicle	1	255
296–306	Transportation – Travelling	1.1, 1.2, 1.3	261
311–313	Cumulative Review and Test		
315–323	Accommodation – Renting and Purchasing	1	265
320–321	Accommodation – Mortgages and Household Insurance	2.1	272
324–329	Accommodation – Building, Decorating and Furnishing	2.1, 2.2, 2.3	276
335–343	Accommodation – Mortgages and Household Insurance	1	271
344–345	Accommodation – Renting and Purchasing	2	276
346–347	Accommodation – Mortgages and Household Insurance	1.4	271
348–349	Accommodation – Renting and Purchasing	1.5	265
350–352	Work – Income	1	227
358–370	Accommodation – Building, Decorating and Furnishing	1	275
373–375	Cumulative Review and Test		
377–402	Work – Income Tax	1	233
406–420	Cost of Independence – Life and Health Insurance	1	287
426–431	Banking – Personal Banking	1.5.3	239
432–438	Investment: Stocks and Mutual Funds		
443–445	Cumulative Review and Test		

REVERSE TEXTUAL CORRELATION
HOUGHTON MIFFLIN CANADA LIMITED
CONSUMER MATHEMATICS

Page	Unit	Learner Expectation Number	TRM Page
3–13	Number Skills Review		
14–16	Time, Tables and Charts, Units of Measure		
18–29	Work – Income	1.1	227
32–43	Work – Income	1.1, 1.4, 1.2.1	227
46–59	Work – Income	1.2, 1.3, 1.4	227
50–53	Work – Income Tax	1	233
62–77	Cost of Independence – Budgeting	2	283
79–82	Work – Income	1	227
84–97	Banking – Personal Banking	1.1, 1.2, 1.3	237
100–107	Banking – Personal Banking	1.1, 1.2, 1.3.1	237
108–115	Banking – Personal Banking	1.4, 1.5	239
119–120	Cumulative Review		
122–135	Banking – Consumer Credit	2	246
140–153	Banking – Consumer Credit	1	243
155	Cost of Independence – Budgeting	1(extension)	279
158–171	Transportation – Purchasing a Vehicle	1	251
164–171	Transportation – Operating a Vehicle	1	255
170–171	Transportation – Travelling	1.2	261
175–178	Transportation – Operating a Vehicle	1	255
175–176	Transportation – Purchasing a Vehicle	1.1.1, 1.2	251
180–187	Accommodation –Mortgages and Household Insurance	1	271
186–189	Accommodation – Renting and Purchasing	1.4, 1.5	265
193–194	Cumulative Review		
196–197	Accommodation – Renting and Purchasing	1.3, 1.5	265
198–201	Accommodation – Mortgages and Household Insurance	2	272
202–215	Accommodation – Renting and Purchasing	1.1, 1.2, 1.5	265
215	Accommodation – Mortgages and Household Insurance	2	272
217–222	Accommodation – Renting and Purchasing	1.3, 1.4, 1.5	265
224–227	Accommodation – Building, Decorating and Furnishing	2.4	276
228–229	Transportation – Operating a Vehicle	1.3	255
230–233	Accommodation – Building, Decorating and Furnishing	2.4	276
236–247	Cost of Independence – Budgeting	2	283
240–241	Accommodation – Renting and Purchasing	1.4	265
244–245	Work – Income	1 (extension)	227
249	Work – Income	1 (extension)	227
252–266	Nutrition		
268–275	Cost of Independence – Budgeting	1	279
277	Banking – Personal Banking	1 (extension)	237
279–280	Cumulative Review		
281–286	Work – Income Tax	1.3	233
288–303	Cost of Independence – Life and Health	1	287
304–317	Banking – Personal Banking	1.5.3	239
318–319	Work – Income	1.2 (extension)	227
324–334	Cost of Independence – Budgeting	1 (extension)	279
336–350	Personal Computers		
351–352	Cumulative Review		
356–388	Basic Number Skills Review		
389–399	Basic Skills: Applications		
400–402	Geometry and Measurement		

APPENDIX A

TEMPLATE: BINOMIAL GRID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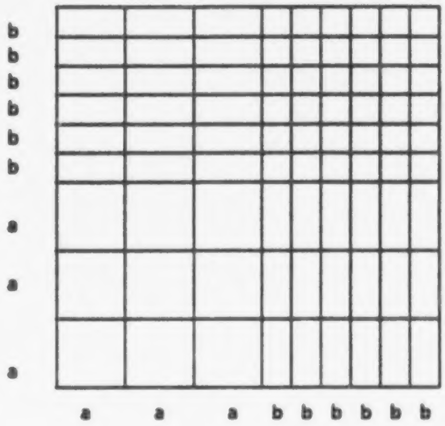
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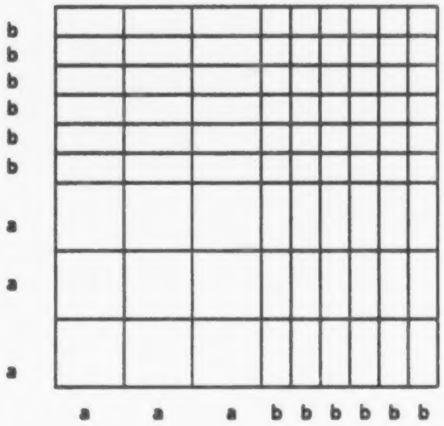
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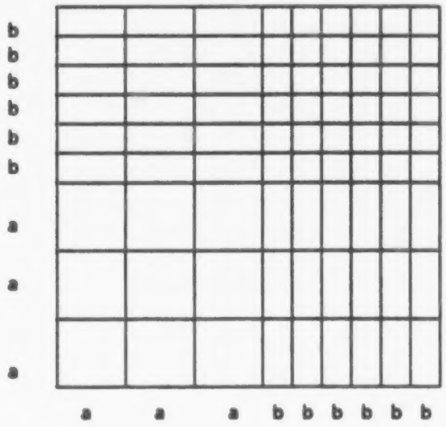
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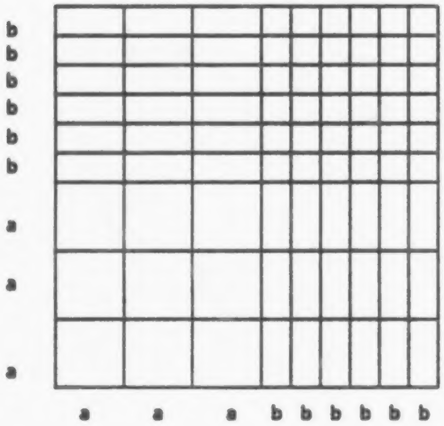
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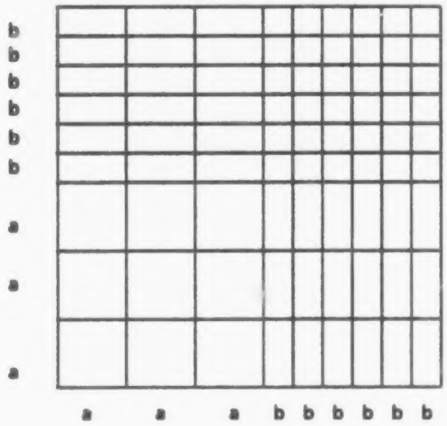
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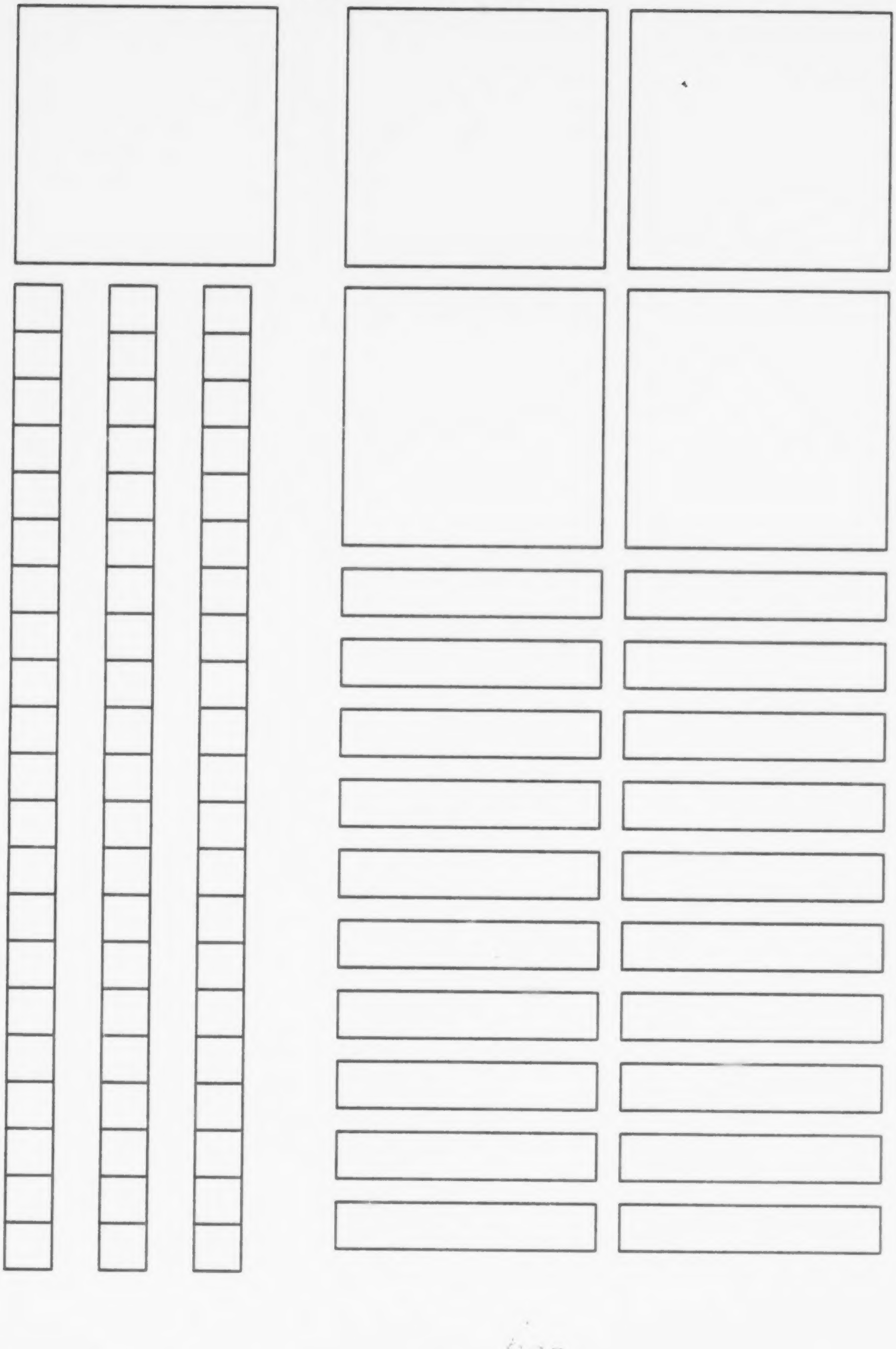
TEMPLATE: ALGEBRA TILES



APPENDIX B

SUPPORT RESOURCES

(A) Print

Activities for Implementing Curricular Themes from the 'Agenda for Action'. (Edited by C.R. Hirsch). Reston, VA: National Council of Teachers of Mathematics. ISBN: 0-87353-229-5. (Student) Provides activities in manipulatives, problem solving, "expanded" basic skills (e.g., exploratory data analysis, estimation), and calculator and computer use.

Activities for Junior High School and Middle School Mathematics: Readings from the 'Arithmetic Teacher' and the 'Mathematics Teacher'. (Edited by K.E. Easterday, et al.) Simpson. Reston, VA: National Council of Teachers of Mathematics, 1981. ISBN: 0-87353-188-4 (Student) Provides activities in a wide range of mathematics topics.

Algebra Tiles for the Overhead Projector. (Howden, H.). New Rochelle, NY: Cuisenaire, 1985. ISBN: 0-914040-42-1 (Teacher) Describes using tiles on the overhead projector to demonstrate polynomials; the zero principle; adding, subtracting and multiplying polynomials; and factoring polynomials.

Algebra Tiles. Student Set.

Art and Techniques of Simulation – Teacher's Edition. (Gnanadesikan, M. et al.) Palo Alto, CA: Dale Seymour Publications, 1987. (Teacher) Provides teachers with ideas for implementing the suggestions identified within the student resource. Offers solutions to student exercises.

Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1989. ISBN: 0-87353-273-2 (Teacher) Provides a discussion on the direction of curriculum and evaluation in mathematics. The document discusses "standards" for curriculum in K-4, Grades 5-8 and Grades 9-12 and "standards" for evaluation of mathematical content, mathematical disposition and mathematical problem solving. There are several teaching and evaluation ideas throughout the resource.

Exploring Probability – Teacher's Edition. (Newman, C.M. et al.) Palo Alto, CA: Dale Seymour Publications, 1987. (Teacher) Provides teachers with ideas for implementing the suggestions identified within the student resource. Offers solutions to student exercises.

How to Evaluate Progress in Problem Solving. (Charles, R. et al.) Reston, VA: National Council of Teachers of Mathematics, 1987. ISBN: 0-87353-241-4 (Teacher) Discusses goals of problem solving evaluation, evaluation techniques, organizing and managing an evaluation program, and using evaluation results.

Mathematics Dictionary. (4th ed.) (James and James.) New York: Van Nostrand Reinhold, 1976. ISBN: 0-442-24091-0 (Teacher) Provides authoritative definitions of mathematics terms.

Problem Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985. (Teacher) Produced as a junior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, a bank of problems, methods of evaluating problem solving performance and a 36-item problem solving bibliography.

Problem Solving in Mathematics: Focus for the Future. Edmonton: Alberta Education, 1987 (Teacher) Produced as senior high school resource. Provides discussion of the nature of problem solving, strategies for teaching problem solving, project and computer related problems, a bank of mathematics problems, methods of evaluating problem solving performance and a 123-item problem solving bibliography.

The Language of Graphs: A Collection of Teaching Materials. (draft). (Swan, M.) Nottingham England: Shell Centre for Mathematical Education, University of Nottingham, n.d. ISBN: 0-906126-11-8 (student) Provides a wide range of graphing activities with emphasis on interpreting for meaning. Topics include modeling and interpreting with points, modeling situations, understanding distance-time graphs and interpreting complex graphs.

(B) Video

Of Dice and Men. (National Film Board) Available through ACCESS NETWORK Media Resource Centre or through the Regional and Urban Film Centres (See Appendix E). One fifteen minute video. Provides a nice introduction into the development of probability and the uses of probability.

Trigonometric Functions I. (TV Ontario) Available through ACCESS NETWORK Media Resource Centre or through the Regional and Urban Film Centres (See Appendix E). A series of six 10-minute vignettes dealing with trigonometric functions. The six topics include: Similar Triangles, Solving Right Triangles, Angles on a Plane, The Sine Law, The Cosine Law and Applications of the Sine Law and Cosine Law. In Math 20, this resource provides a good review of the trigonometric functions (the first two vignettes) and then provides a good introduction into the angles on a plane and the sine and cosine laws. In Math 23, the first two vignettes will provide an alternative form of review when the unit is complete.

(C) Computer Software

Computer Graphing Experiments I. Addison-Wesley, (Apple II) Capable of graphing linear equations, parabola exponential-logarithm functions, inverse functions special functions such as absolute value and polynomials. "Linear Equations" option provides following sub-options: graphs of the form $y = mx$ and $y = mx + b$, and simultaneous equations. Students specify equation by inputting m and b. Extensive student worksheets provide activities in investigating effects of ma and b, parallel and perpendicular lines, writing equation in $y = mx + b$ form given $Ax + By + C = 0$ form, two points or point-slope. Point of intersection of system of equations identified as ordered pair. Graphs of straight lines slightly ragged, appearance of perpendicularity problematic, no grid. Suitable for teacher demonstration, small group and individual student use. Teacher's guide with extensive set of student activity worksheets.

Master Grapher. (Waits, B. et al.) Reading, Mars: Addison-Wesley, 1988. (Apple II, IBM PC or PS/2, Macintosh) ISBN: 0-201-18246-7. Versatile utility graphing program capable of graphing a variety of functions, conics, parametric equations and polar form relations. Linear equations can be graphed in "Function Grapher" option. Suitable for exploring effects of m and b in $y = mx + b$, graphing systems of linear equations. Features include sliding vertical and horizontal lines, or cross-hairs that permit identification of x- and y-intercepts and point of intersection, change of viewing window, zoom in/out, grid display. Suitable for teacher demonstration, small group and individual student use.

Quantitative Literacy Software. American Statistical Association (Apple, IBM). Software to be used as a tool in examining statistics. The software provides support the Quantitative Literacy Series of resources: Exploring Data, Exploring Probability, Art and Techniques of Simulation and Exploring Surveys. This software is particularly useful at the Math 20 and 23 levels as it takes you through many simulations for probability.

SRA Computer Drill and Instruction, Mathematics, Level D. Willowdale, On: Science Research Associates, 1981–1983. (Apple II, Recommended resource at the Junior High Level. Also available for IBM PC) A multi-disk package of drill exercises in computation, numbers and numeration, fractions, decimals, ratio and percent, measurement, pre-algebra and applications.Level appropriate for various copies in Mathematics 14, some in Mathematics 13. Does not provide answers or method of solving. May be used by students in unmanaged or teacher-managed form.

OTHER RESOURCES

(A) Computer Software

Algebra Drill and Practice I. (Detmer, R. et al.) Iowa, City, IA: Conduit, 1981. (Apple II, IBM PC) Available from Conduit, University of Iowa, Oakdale Campus, Iowa City, IA 52242. $125 US (1988) Two-disk package provides drill and practice in a variety of topics including algebraic fractions, slopes and equations of line, percentages, simplifying linear algebraic expressions and word problems. "Slopes and equations of lines" option offers four types of problems: find slope given two points or given the equation in $Ax + By + C = 0$ form and find equation of line given point and slope or two points. Must be expressed in $Ax + By + C = 0$ form. "Percentage" option give percent problems of the form "20 is 30% of what number?" Provides example solutions, option of seeing solution to current problem. Typically, student inputs answer only, must solve mentally or on paper first. Teacher's guide included.

Algebra Drill and Practice II. (Detmer, R. et al.) Iowa, City, IA: Conduit, 1982. (Apple II, IBM PC) Two-disk package provides drill and practice in simplifying integral exponent expressions, solving absolute value problems, linear equations and linear systems in two variables, factoring binomials and trinomials, finding roots of quadratic equations and solving linear inequalities. Step-by-step presentation of solution of examples and problems with which student has difficulty provided. Student inputs answer only, must solve mentally or on paper. Optional graphic solution of linear systems and inequalities provided by program following successful algebraic solution by student. Teacher's guide included.

Coordinate Math. St. Paul, MN: Minnesota Educational Computing Corporation, 1987. (Item No. A-192.ISBN: 0-87490-399-8. Apple II. Available from MECC, 3490 Lexington Avenue North, St. Paul, MN 55126) Consists of three educational games that require students to use their skills at problem-solving and locating and using coordinate points on the Cartesian plane to accomplish a stated goal. Teacher's guide, with student worksheets, included.

Equation Math. St. Paul, MN: MECC, 1987. (Item No. 1-193. ISBN: 0-87490-405-6. Apple II) A versatile graphing program that offers three main options: "Exploring Equations," "Polygraph" and "Polar." The "Exploring Equations-Linear" option provides opportunity to explore effects of ma and b in $y = mx + b$. Equations of form $x = b$ and $y = b$ acceptable. Hard copy of screen contents optional Automatic increment/decrement of parameters through cursor key action provided. "Quadratic equation" option also available.. "Polygraph" option capable of graphing numerous types of relations, including linear. Zoom in/out option provided. Display of ordered pairs that satisfy relation optional. Suitable for teacher demonstration and small group or individual student use. Teacher's guide, with extensive set of student worksheets, included.

Equations. (Ross, D.) Northbrook, IL: Mindscape, 1982. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. Available form Mindscape, Inc., 3444 Dundee Road, Northbrook, IL 60062. $29.95 US (1988)). Provides very structured practice in solving equations of the form $AX + B = C$. Format is interactive step-by-step solution of the problem presented. Presentation of example solution and solutions of problems with which student has difficulty provided.

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Equations II. (Appel, M. et al.) Northbrook, IL: Mindscape 1984. (Apple II, IBM PC, Commodore 64/128,Tandy 1000.Available form Mindscape, Inc., 3444 Dundee Road, Northbrook, IL 60062. $29.95 US (1988)).Provides very structured practice and remedial opportunities in solving equations of the form $Ax + B = Cx + D$. See Equation.

Equations III. (Ross, D.) Northbrook, IL: Mindscape, 1985. (Apple II) Provides very structured practice in solving equations of the form $Ax + B(Cx + D)\text{-}Ex + F$. See Equations.

Factoring Algebraic Expressions. Northbrook, IL: Mindscape. (Apple II, $29.95 US (1988)). Provides practice and remedial opportunities in factoring common factor problem, trinomials ("easier" :$a = 1$ and harder :$a \neq 1$) and difference of squares Students must determine factors mentally or on paper, input as directed on screen. Program provides check by multiplication. Optional presentation of solution available.

Geometric preSupposer: Points and Lines. (Schwarta, J. et al.) Scarborough, ON: Sunburst, 1986. (Apple II. Available from Sunburst Communication, P.O. Box 3240, Station F, Scarborough, ON, M1W 9Z9. $132 (1989) Provides extensive possibilities for drawing points, lines, segments, triangles, quadrilaterals, polygons, circles, bisectors of angles and parallel and perpendicular lines. Measurement and/or calculation of length, perimeter, circumference, area or angles possible. Lines tend to be slightly ragged. Will take time for students to develop facility with program. Suitable for teacher demonstration, or geometry exploration by small group or individual students. Extensive teacher's guide, with student worksheets, included.

Geometric Supposer: Triangles. (Schwartz, J. et al.) Scarborough, ON: Sunburst, 1985. (Apple II, IBM PC, Tandy 1000) Provides extensive possibilities for exploring properties of triangles, which may be created by angle or side classification, or by construction according to SSS, SAS or ASA. Other figures such as line segments, circles, medians, altitudes, parallel and perpendicular lines, bisectors of angles and perpendicular bisectors of lines may be constructed. Measurement of lengths, perimeters, areas, angles and distance possible. Sum, difference, product, square and ratio of various measures also possible. See other comments, "Points and Lines." Extensive Teacher's guide, with student worksheets, included.

Graphing Linear Functions. (Berman, M. et al.) Northbrook, IL: Mindscape, 1984. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. $29.95 US (1988)). Provides structured practice in graphing equations of the form $y = mx + b$ ("Easier" option) and "Harder" forms, for example, $Ax + By + C = 0$, $By = Ax + C$, $(B \neq 1)$, etc. Students then determine three sets of ordered pairs that satisfy equation and plot by means of cursor key movements. Program then draws line in "Harder" forms option, students must first solve for y in step–by–step fashion.

Green Globs and Graphing Equations. (Dugdale, S. et al.) Scarborough, On: Sunburst. 1986. (Apple II, IBM PC, Tandy 1000. $87 (1989)). A versatile graphing program. "Equation Plotter" option is a utility program in which a wide variety of forms of equations can be input and graphed. "Linear" sub–option provides opportunity to explore effects of m and b in $y = mx + b$; input in $Ax + By + C = 0$ form; investigate parallel and perpendicular lines and $y = b$, $x = b$ forms; etc. Systems of equations may also be graphed, although coordinates of intersection point not given. Other modes of "Equation Plotter" permit such things as rational, polynomial, absolute value quadrated and exponential functions and identifying zeros. Suitable for teacher demonstration and small group or individual student use. "Linear and Quadratic Graphs" option provides practice in writing equations. Program draws graph, student must write equation that satisfies graph. Extensive teacher's guide, with student worksheets, included.

Percents. Baltimore: Media Materials, 1983. (Apple II, except II c). Available from Media Materials, 2936 Remington Avenue, Baltimore, MD 21211. $49.95 US (1988)). Provides practice and remedial opportunities in solving percent problems. Teacher's guide, with student worksheets, included.

Ratios and Proportions. Baltimore: Media Materials, 1983. (Apple II, except II c). Provides practice in solving ratio and proportion problems. Teacher's guide, with student worksheets, included.

Solving Quadratic Equations. (Ross, D. et al.) Northbrook, IL: Mindscape, 1982. (Apple II, IBM PC, Commodore 64/128, Tandy 1000. $29.95 US (1988)). Provides structured practice in solving quadratic equations. Two options: "Easier (A = 1)" and "Harder (A = 2 or 3)." Students presented with quadratic equation, which must be factored mentally or with pencil-and-paper. Program checks by multiplying. Students then must solve for each of the factors in step-by-step manner. Program demonstrates solution for questions student has difficulty with.

Spreadsheets for Mathematics and Science: Appleworks Sampler. St. Paul, MN: MECC, 1987. (Item No. T-260. ISBN: 0-87490-415-3, Apple II) Provides practice in learning and using spreadsheets (specifically, Appleworks) through its application to mathematics and science problems. Requires time. Teacher must know how to use spreadsheet before introducing to students. Extensive teacher's guide, with student activity sheets, included.

(B) Print

A Sourcebook of Applications of School Mathematics. Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1980. ISBN: 0-87353-164-7. (Teacher) Contains a number of articles that discuss use of applications in mathematics, and provide specific examples of application in advanced arithmetic, algebra, geometry, trigonometry and probability.

Activities from the 'Mathematics Teacher'. (Edited by E.M. Maletsky et al.). Reston, VA: National Council of Teachers of Mathematics, 1981. ISBN: 0-87353-173-6. (Student) Provides activities in a number of mathematics topics.

Algebra Before & After: 1. (Gregory, J.) Palo Alto, CA: Creative Publications, 1987. ISBN: 0-88488-641-7 (Student) A workbook of student developmental and practice activities related to topics in polynomials (exponents, writing expressions, multiplying and factoring), writing and solving linear equations and systems of equations, coordinate geometry and graphing, square roots and the Pythagorean Theorem. Emphasis is on pattern development and generalization.

Algebra Before & After: 2. (Gregory, J.) Palo Alto, CA: Creative Publications, 1987. ISBN: 0-88488-642-5 (Student) A workbook of student developmental and practice activities related to topics in radicals and rational exponents, polynomials (multiplying, factoring, number pattern generalizations) rational expressions, relations and functions (graphing, zeros), and quadratic equations. Emphasis is on pattern development and generalization.

Applications in School Mathematics: 1979 Yearbook. (Edited by S. Sharron et al.) Reston, VA: National Council of Teachers of Mathematics, 1979. ISBN: 0-87353-139-6 (Teacher) Numerous examples provided of applications in mathematics in this collection of articles. Extensive bibliography also provided.

Canada's Food Guide Handbook. Ottawa: Department of National Health and Welfare, 1988. ISBN: 0–662–11947–9 (Teacher) Provides resource material suitable for use in ratio and proportion problems and projects.

Classroom Ideas from Research on Secondary School Mathematics. (Dessart, D. & Suydam, M.) Reston, VA: National Council of Teachers of Mathematics, 1983. ISBN: 0–87353–207–4. (Teacher) Presents a variety of ideas for teaching algebra and geometry in the secondary school. Discusses research on teaching, cognitive development, attitude and affective factors, problem solving, the use of calculators and computers, and testing and prediction.

Computers in Mathematics Education, 1984 Yearbook. (Edited by V.P. Hansen et al.) Reston, VA: National Council of Teachers of Mathematics, 1984. ISBN: 0–87353–210–4. (Teacher) Discusses computing from standpoints of its challenge, impact and perspective as a diagnostic tool. Discusses programming as a means of teaching mathematics.

Computing and Mathematics: The Impact on Secondary School Curricula. (Edited by J.T. Fey.) Reston, VA: National Council of Teachers of Mathematics, 1984. ISBN: 0–87353–212–0. (Teacher). Report of a major conference. Topics include mathematics and the new information technology, impact of computing on algebra, geometry, calculus and discrete mathematics and algorithmic methods and prospects and strategies for change in school mathematics.

Creative Geometry: Investigations on the Microcomputer. (Teacher's Guide) (Bell, A. et al., South Notts Project) Nottingham, U.K.: Shell Centre for Mathematical Education, University of Nottingham, University Park, Nottingham, NG7 2RD, n.d. Discusses activities in creating geometric patterns and designs by creating and using programs like those in LOGO.

Descartes' Dream: The World According to Mathematics. (Davis, P. et al.) Boston: Houghton Mifflin, 1986. ISBN: 0–395–43154–9. (Teacher) Discusses the nature of mathematics, the impact of computing on mathematics, the impact of both on society. Eclectic in style, highly readable.

Estimation and Mental Computation: 1986 Yearbook. (Edited by H.L. Schoen et al.). Reston, VA: National Council of Teachers of Mathematics, 1986. ISBN: 0–87353–226–0 (Teacher) A collection of articles that provide rationale and strategies for developing and evaluating estimation and mental computation skills.

Everybody Counts: A Report to the Nation on the Future of Mathematics Education. (Board on Mathematical Sciences [and] Council (U.S.)) Washington, D.C.: National Academy of Sciences, 1989. ISBN: 0–309–03977–0 (Teacher) Discusses mathematics, mathematics education and their relation to society. Relevant to Canadian educators.

Imaginative Ideas for the Teacher of Mathematics, Grades K–12: Ranucci's Reservoir. (Ranucci, E.R., Edited by M.A. Farrell.) Reston, VA: National Council of Teachers of Mathematics, 1988. ISBN: 0–87353–257–0 (Teacher) A variety of topics covered in this collection of articles. Often suitable for extension activities.

Learning and Teaching Geometry, K–12: 1987 Yearbook. (Edited by M.M. Lindquist et al.) Reston, VA: National Council of Teachers of mathematics, 1987. ISBN: 0–87353–235–X (Teacher) Contains a number of articles that deal with topics such as issues in teaching geometry, problem solving activities and relating geometry to other mathematics topics.

Math Projects for Young Scientists. (Thomas, D.A.) New York: Franklin Watts, 1988. ISBN: 0-531-10523-7 (Student) Discusses how to choose, research and present a mathematics project, and briefly describes 96 suggested student projects. Designed for the capable, interested student of mathematics. Varying degrees of background mathematics sophistication required.

Mathematics: A Human Endeavour. (2nd ed.) (Jacobs, H.R.) New York: W.H. Freeman, 1982. ISBN: 0-7167-1326-8. (Student) An innovative textbook exploring a variety of mathematics topics.

Mind Tools: The Five Levels of Mathematical Reality. (Rucker, R.) Boston: Houghton Mifflin, 1987. ISBN: 0-395-46810-8 (Teacher) Explores the nature of mathematics as number, space, logic, infinity and information. Very non-routine but readable discussion of the nature of mathematics.

Mindstorms: Children, Computer, and Powerful Ideas. (Papert, S.) New York: Basic Books, 1980. ISBN: 0-465-04629-0 (Teacher) Describes the development of LOGO and its potential use by children to develop mathematical thinking and understanding.

Problem Solving in School Mathematics: 1980 Yearbook. (Edited by S. Krulik et al.). Reston, VA: National Council of Teachers of Mathematics, 1980. ISBN: 0-87353-162-0. (Teacher) Contains a number of articles describing various aspects of problem solving.

Recent Revolutions in Mathematics. (Stwertka, A.) New York: Franklin Watts, 1987. ISBN: 0-531-10418-4 (Teacher) Provides a readable overview of recent activity in the field of mathematics. Topics include the geometry of the universe, the science of prediction, number theory, fractals, chaos and catastrophe theory and computers.

Student Math Notes. (Edited by E.M. Maletsky) Reston, VA: National Council of Teachers of Mathematics, 1987. ISBN: 0-87353-244-9. Contains a number of articles with interesting non-routine student activities. See for example: "Pi: The Digit Hunt" (Coffield, P.), pp. 39-44. Presents a number of historical notes and activities regarding the approximation of pi.

Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies. (2nd ed.) (Sobel, M. et al.) Englewood Cliffs, NJ: Prentice Hall, 1988. ISBN: 0-13-894148-3 (Teacher) A sourcebook for secondary mathematics teachers. Covers a wide variety of topics.

Teaching Statistics and Probability: 1981 Yearbook. (Edited by A.P. Shulte et al.). Reston, VA: National Council of Mathematics, 1981. ISBN: 0-87353-170-1 (Teacher) Provides discussion on rationale, samples of existing programs, classroom activities, teaching and learning specific topics, applications, statistical inference, Monte Carlo techniques, using computers and an extensive bibliography and list of projects.

The Art of Problem Posing. (Brown, S.I. et al.) Philadelphia: Franklin Institute, 1983. ISBN: 0-89168-052-7 (Teacher) Provides strategies for and numerous examples of problem posing, designed to encourage open-ended problem solving.

The Ideas of Algebra, K-12: 1988 Yearbook. (Edited by A.F. Coxford et al.). Reston, VA: National Council of Teachers of Mathematics, 1988. ISBN: 0-87353-250-3 (Teacher) Covers a number of topics related to school algebra, including ideas and issues, readiness, equations and expressions, problem solving, computer and calculator use, and teaching ideas.

The Mathematical Experience. (Davis, P. et al.) Boston: Birkhauser, 1981. Penguin Books ed. (1983) ISBN: 0-14-022456-4. (Teacher) Explores widely the nature of mathematics: history, schools of philosophical thought, symbolic nature, views of some contemporary mathematicians and teachers, teaching and learning and uses. Insightful, written in highly readable style.

The Nature of Mathematics. (4th ed.) (Smith, K.J.) Monterey, CA: Brooks/Cole, 1984. ISBN: 0-534-02806-3 (Student) Innovative, wide-ranging textbook. Explores the nature of a number of mathematics topics, such as inductive and deductive reasoning through patterns and sets, calculators and computers, numbers, algebra, geometry, and statistics and probability.

Thinking Mathematically. (rev.ed.). (Mason J. et al.) Wokingham, Eng.: Addison-Wesley, 1985 ISBN: 0-201-10238-2 (Teacher) A structured, activity-based book of discussion and open-ended questions designed to help develop ability to think mathematically. Attends to the emotional aspects of working with mathematical problems. Very readable and engaging.

(C) Journals

Computers in Education. Moorhead Publications, Ltd., 1300 Don Mills Road, North York, Toronto, ON, M3B 3M8. (Teacher) Provides information on microcomputers and software. Canadian focus. Articles, items discuss computing and programming uses in education, including mathematics education. Published 10 times a year.

delta-K. Mathematics Council of the Alberta Teachers' Association. (Teacher) (Address correspondence to J.B. Percevault, editor, 2510 - 22 Avenue South, Lethbridge, Alberta; T1K 1J5 Articles include discussions of issues in mathematics education, general approaches and styles in mathematics teaching and learning, and specific teaching ideas.

delta-K. Special Edition: "Mathematics for Gifted Students." Vol. 27, No. 3, April 1989. Presents articles on special topics in mathematics, plus lists of more than 350 problems with solutions and more than 140 books of mathematical puzzles, problems, instruction and applications.

For the Learning of Mathematics. FLM Publishing Association, 4336 Marcil Avenue, Montreal, Quebec H4A 2Z8. (Teacher) Articles deal with a variety of mathematics-related topics, including issues in mathematics education, philosophy of mathematics and research on teaching and learning mathematics. Intended for a wide range of mathematics educators. Published three times a year.

Mathematics Teacher. National Council of Teachers of Mathematics. 1906 Association Drive, Reston, Va 22091 (Teacher) A journal intended for the secondary mathematics teacher. Scope includes mathematics education issues, teaching ideas, innovative secondary level mathematics and resource reviews. Published 10 times annually.

School Science and Mathematics. School Science and Mathematics Association, Inc., 126 Life Science Building, Bowling Green University, Bowling Green, OH 43403. (Teacher) Articles discuss issues in mathematics and science education, research on teaching and learning mathematics, science and teaching ideas.

The Computing Teacher. International Council for Computers in Education, University of Oregon, 17787 Agate Street, Eugene, OR, 97403-9905 (Teacher) Emphasizes teaching and learning about computers, teaching with computers, teacher education and the impact of computers on curricula. Published nine times a year.

APPENDIX C

SAMPLES OF STUDENT EVALUATION IDEAS

Evaluating Problem Solving

Rating	Exceptional (5)	Proficient (4)	Satisfactory (3)	Limited (2)	Poor (1)
Category					
Understands problem.	Complete understanding of problem.	Most of problem understood.	Part of problem understood.	Limited understanding of problem.	Complete misunderstanding of problem.
Develop plan to solve problem.	Plan would lead to a logical conclusion.	Mostly appropriate plan; may have to be revised.	Partially appropriate plan; based on an incomplete understanding of problem.	Limited attempt to develop a plan.	No attempt to develop a plan; no logical conclusion could be reached.
Arrives at a solution.	Logical solution to problem; appropriate answer.	Solution mostly appropriate; minor error.	Some error in answer, but solution direct result of error in plan.	Major error in solution.	No answer given; illogical solution.
Evaluates solution.	Solution evaluated accurately based on information provided.	Evaluation mostly complete and appropriate.	Satisfactory attempt to evaluate solution and use data collected.	Limited attempt to evaluate solution or use data.	No attempt to evaluate solution or use data collected.

Note: Student excused –
Unexcused absence – rating 0
Assignment not done or handed in – rating 0

Science Modules: Teacher Resource Guide

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Checklist

Student: **Date:**

_____ 1. Likes to solve problems
_____ 2. Works cooperatively with others in the group
_____ 3. Contributes ideas to group problem solving
_____ 4. Perseveres – sticks with a problem
_____ 5. Tries to understand what the problem is about
_____ 6. Can deal with data in solving problems
_____ 7. Thinks about which strategies might help
_____ 8. Is flexible – tries different strategies if needed
_____ 9. Checks results for accuracy
____ 10. Can describe or analyse results – come to an appropriate conclusion or decision

Rating Scale

Student: **Date:** ______________________________

	Frequently	Sometimes	Never
1. Selects appropriate solution strategies	_____	_____	_____
2. Accurately implements solution strategies	_____	_____	_____
3. Tries a different solution strategy when stuck (without help from the teacher)	_____	_____	_____
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, plans, solves and checks)	_____	_____	_____
5. Shows a willingness to try problems	_____	_____	_____
6. Demonstrates self-confidence	_____	_____	_____
7. Perseveres in problem-solving attempts	_____	_____	_____

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	Student 1	Student 2	Student 3	Student 4	Student 5
Date: Name Action observed					
Confidence: Initiates questions.					
Is sure answers will be found.					
Helps others with problems.					
Other/note:					
Flexibility: Solves problems in more than one way.					
Changes opinion when given a convincing argument.					
Other/note:					

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SELF-ASSESSMENT DATA FROM STUDENTS

Rating Scale

Use the following questions to help you look back and describe your thinking as you worked toward a solution to the problem (or worked through the investigation, assignment, project).

1. What did you do when you first saw the problem? What were your thoughts?
2. Did you use any problem-solving strategies? Which ones? How did they work?
3. Did you try one approach that didn't work and have to stop and try another approach? How did you feel about this?
4. Did you complete the assignment? (investigation, project) How do you feel about this?
5. Did you check your work in any way to assure the original assignment was carried out correctly? Did you feel sure it was correct? (satisfactory)

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FOCUSSED MARKING SCALE

7. EXCEPTIONAL

   - content: ideas, organization and development command attention and respect
   - an impressive style evident from word choice, usage and sentence structure
   - mechanical skill accurate and effective in relation to purpose
   - accurate spelling

6. SUPERIOR

   - content: most ideas are significant; well organized and developed
   - sentence structure and word choice clear and effective; accurate usage
   - mechanical skills relatively error free in relation to purpose
   - relative free from spelling errors

5. COMPETENT

   - content: several significant ideas; minor problems in organization and development
   - minor problems in word choice, and/or sentence structure
   - frequent mechanical errors
   - few spelling errors

4. MARGINAL

   - content: a few relevant ideas; some evidence of organization but with deficiencies in development
   - many flaws in word choice and/or sentence structure
   - many distracting mechanical errors
   - numerous, distracting spelling errors

3. UNSATISFACTORY

   - content: insufficient number of ideas; disorganized, inadequate development
   - many flaws in word choice and/or sentence structure
   - many distracting mechanical errors
   - numerous, distracting spelling errors

2. MAJOR DEFECTS

   - content: lacking in ideas, or organization and/or development
   - serious deficiencies in word choice and/or usage and/or sentence structure
   - extensive mechanical errors
   - extensive spelling errors

1. INSUFFICIENT DATA AND/OR OFF TOPIC

Developed by Program Evaluation and Language Arts Team, Calgary Board of Education.

Suggestions for Further Reading

Board of Education for the City of Etobicoke (1987). Making the Grade: Evaluating Student Progress. Prentice-Hall Canada Inc. Scarborough Ontario.

Joint Mathematical Board Shell Centre for Mathematical Education (undated). Problems with Patterns and Numbers: An O-level Module.

National Council of Teachers of Mathematics (1988). How to Evaluate Progress in Problem Solving. Reston Virginia.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston Virginia.

APPENDIX D

USING WRITING TO LEARN MATHEMATICS

C.L. Nahragang and B.T. Petersen

Writing has a place in a mathematics class. It provides a new and interesting mode of communication between the student and teacher, as Watson (1980) reported:

> *This two-way conversation has become beneficial to the class. The students realized I hear them and care. They seem to have looked inside themselves and to have seen what they could do to help solve their mathematical problems. Many of their grades improved.*

Written assignments in mathematics classes also afford students the opportunity to organize their thoughts and, at the same time, improve their writing skills (Johnson 1983.)

The purpose of this article is to show that writing can be used in mathematics to enhance learning. The act of writing gives students the opportunity to formulate, organize, internalize, and evaluate concepts. Emig, who has done extensive investigation of the thinking processes involved in writing, argues that writing "represents a unique mode of learning - not merely valuable, not merely special, but unique" (Emig 1977.) The idea that writing is a powerful aid to learning has also found much support in cognitive psychology. For example, Bruner (1966) states, "the more we know about the manner in which it [writing] can aid thought...[the more it] leads me to put language at the center of the stage in considering the nature of intellectual development."

Our experience indicates that the most effective method of using writing to help students learn mathematics is through the use of journals. These in-class writing exercises offer students the opportunity to work informally and personally on mathematical concepts, using their own language and real-world experiences. They can plan with a variety of concepts without the fear of the formal evaluation of their writing and their mathematical skills. Our experience in college -level classes, from algebra through calculus, makes us feel confident that these short but powerful writing exercises enhance learning. The journal goes beyond rote learning and challenges the students to use intellectual skills. Myers relates writing to individual learning: "When doing this simple activity [writing], students will be forced to relate information from the lecture to what they already know and to organize and synthesize it so that the concept becomes their own" (Myers 1984.)

The Journal

The journal is a diary-like series of writing assignments. Each assignment or entry is a short written response to an instructor's question, statement, or set of instructions. All responses are written in prose rather than in the traditional mathematical style of numbers and equations. The students are directed to use an expressive writing style so that their responses are spontaneous and all their thoughts are recorded. Figure 1 is an example of a journal entry made by a student in technical writing in response to the following assignment: "Describe a real life, nonmathematical analogy to the rule $a^{-n} = {}^{1}/a^{n}$."

Journals have two basic functions. First, they allow students to proceed at their own rate and to converge on an understanding of mathematical concepts using their own experiences. Second, they provide teachers a unique diagnostic tool; the writings of students immediately reveal areas of confusion and expose misunderstandings of mathematical concepts.

Journals were introduced into our mathematics classes at Michigan Tech with a minimum of difficulty. These classes were conducted over a ten-week period. Each class included about ten unannounced quizzes that accounted for 20 percent of a student's final grade. By substituting twenty journal-writing sessions for these quizzes, we gave our students two opportunities a week for writing in their journals. However, the flexibility - one of its advantages - of the journal-writing technique permits the instructor to devote extra assignments to more difficult mathematical concepts.

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$a^{-n} = \frac{1}{a^n}$ is an inverse rule so, if any power is raised to the neg. it becomes

a fraction, also making it positive.

You inverse what it says, you switch it around.

So what in life is switched around or inversed (which means the same thing?)

What can I turn around in my life - yet keep it the same; to mean the same? (yet it is non–mathematics)

- *relationships*
- *feelings*
- * *communication – the way I say something or communicate*

The english language and communication could easily apply to this rule. We may change or turn phrases around yet keep their meaning the same.

Figure 1

Most journal entries require three to seven minutes, depending on the type of assignment. Our experiences have shown that as students become more familiar and comfortable with this technique, their responses become better organized and thus easier to read. In all classes, we found that students' experiences in writing in journals led to definite improvements in their ability to organize their responses.

The method of evaluating journals become an important question. Initially, grading each journal seemed to be most appropriate, with the average of all journal grades being a part of the final grade for the course. We immediately realize that this method restricted the potential of this learning tool. By assigning grades, we were indirectly telling the students how they should process concepts, which was defeating one of the important attributes of journal writing – individualized learning and discovery. However, students seemed to deserve some type of credit for their efforts. After considering several alternatives, we decided to give one or two points toward the next major examination on the basis of the students' comments about concepts., The most appropriate responses to mathematical concepts were given the maximum extra credit allowable. A lack of effort received no credit. In addition, we often added appropriate comments (e.g., "excellent," "good," "incomplete," or "please revise.") We found that students appreciated the teacher's comments as much as they did the extra credit. Finally, all mathematical errors were corrected. Although these errors were a secondary consideration, they were brought to the students' attention.

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An Analysis

Journal entries demonstrate individual learning activities. Various types of assignments were made that coincided generally with the levels specified in Bloom's taxonomy (Bloom, Hastings, and Madaus 1971.) During a section on factoring, students were asked to do the following journal assignment: "Discuss the following statement: 'Factoring and finding a product are reverse processes.'" The objective of this particular assignment was to have students collect ideas, internalize the concept of factoring, and examine its significance in finding a product. This task was assigned early in the study of factoring and finding products of polynomials. Observe how each of the following students begins to understand the process of factoring in a personal frame of reference. Student A wrote:

Factoring and finding a product are reverse processes. They result as reverse processes but the way you go about it isn't reverse because the opposite of multiplication is division. The equations are interchangeable though and can be called.

At first, this response appears to be nonsense. Notice, however, the student's attempt to link multiplication ad division to finding a product and factoring. This link is important because it gives this student a bridge to familiar concepts, multiplication and division.

Even in the response of student B, a marginal student, we can see the cognitive value of writing:

Factoring and finding a product are reverse processes. Factoring I guess is to make finding the answer much easier. I really am not sure how much it helps because I can't see much point in it. I guess that it puts the problem in simpler terms.

First, the student restates the assignment and then works toward a basic but correct evaluation. The importance of this student's response can be seen in the last sentence. The student discovers one of the main purposes of factoring, to simplify an expression so that it can be used in further operations.

Student C shows an understanding of the mechanics of the two processes but demonstrates no insight into the concept or purpose of factoring.

I believe that factoring and finding a product are reverse procedures. When you find a product of two binomials that is multiply:

$(x + 2)(2x - 1) = 2x^2 + 3x - 2$
When you factor a trinomial you end up with two binomials:

$2x^2 + 3x - 2 = (x + 2)(x - 1)$

This response also illustrates a difficulty with evaluating statements. The more equations and mathematics we see in a response, the less we know about students' understanding of a concept. For this reason we always encourage our students to answer with words and sentences rather than with equations.

In the following example, notice how student D collects ideas on finding a product and factoring, then, in the third sentence, attempts to answer self-generated questions, with a satisfactory conclusion.

Combining factors to find a product is an application of algebra that allows factors to be combined into either monomials, polynomials, or trinomials, or whatever. Factoring is the process of taking monomials, polynomials, binomials, trinomials, or whatever and reducing them to the terms that multiply together to form them. Thus, factoring is taking polynomials apart, while finding the product is putting them together. Actually the two are very similar in the processes used. One uses primarily multiplication, the other division.

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The student collects, internalizes, and finally evaluates. For the student, the journal is a progression toward the understanding of a concept. For the teacher, the journal is a record of a student's thinking and thus an excellent diagnostic tool.

Given the same assignment, some students focused their thoughts on isolating a common factor. For this method of factoring, students sometimes confused division with the isolation of a common factor. They did not grasp the idea that division is actually contained within the process of isolating a common factor. The success or failure of students in recognizing this difference is clearly demonstrated by the responses of students E and F, respectively:

Factoring is sort of dividing ($3x^2 + 6x$) when you factor out the 3x you get $3x(x + 2)$. To multiply $3x(x + 2)$ you do the opposite and you get $3x^2 + 6x$ again. So finding a product and factoring are the same simply reversed.

When factoring you remove all common factors from an equation. These are removed usually by dividing them out of the whole equation. Now to get back to the original equation from the factored one, you have to multiply the factored equation by the numbers and varible, etc., that were removed. This shows that factoring and finding the product....

Student E doesn't separate the two ideas as well as student F. In the second sentence student F clearly understands that division is a part of the process of isolating the common factor.

A subsequent assignment in the same class required students to translate the concept of finding a product and factoring to a nonmathematical event. Students responded to the following prompt, "Think of a nonmathematical relationship that is analogous to the process of finding a product and factoring." Student G gave one of the better responses:

One relationship similar to finding a product vs. factoring is that of taking a carburetor apart and then putting it back together. When the carburetor is together it is difficult to clear and repair. Therefore it is disassembled to make it easier to work with. This is analogous to factoring.

When the parts of the carburetor are in working condition, they must be put back together before they will work as desired. This is analogous to finding a product.

This journal assignment differs from the other assignments because students need to compare nonmathematical experiences to what is being learned in class. The response of student G demonstrates an overall understanding of factoring and finding a product by comparing these processes to the disassembly and assembly of a carburetor. He understands their relationship to each other and is able to connect the two processes. His analogy also illustrates that factored and product forms have different mathematical uses. In this assignment the student had to collect information from both mathematical and nonmathematical experiences and then connect them logically. This example shows that in one short journal a student can use a variety of intellectual skills, such as synthesis, interpretation, translation, analysis, and evaluation. We believe that with continued use of these skills, students grow both in the understanding of mathematical concepts and in the ability to express that understanding.

Conclusion

We adapted the technique of journal writing to instruction in mathematics classes. We attempted to promote an improved internalization of mathematical concepts and to see how journal writing could be incorporated in these classes. This study examined students' test scores, attitudes, and writing (the latter by the Writing Apprehension Test) as well as student and teacher evaluations. The data have not established any strong relationships among the various attributes examined. An examination of journals, however, clearly indicates that students used their journals to think about solving problems

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associated with mathematical concepts. In addition, most of the students' responses to a questionnaire indicated that students viewed journals as a worthwhile inclusion in mathematics classes.

Any conclusion derived from this study must be viewed and extended cautiously. They are specific to a particular isolated setting. The study has led to guarded conclusions and has generated many more questions than it has answered. The investigating team hopes that these efforts are more of a "first step" rather than a "stopping place" and that further study will establish more definitive relationships between writing and mathematics.

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EXPLORATIVE WRITING AND LEARNING MATHEMATICS

S.Z. Keith

Short explorative writing assignments can transform the mathematics classroom into a dynamic and exciting learning laboratory. In explorative writing, students explore their knowledge about a topic by writing what they know about it in their own language. Then, to refine their ideas further, they share reactions with other students and the teacher. In spite of the significant benefits of using such writing assignments, the teacher may feel uncomfortable with the more interactive environment, with the challenge of designing writing assignments, and with figuring out what to do with the results. This article illustrates how explorative writing assignments can help to expose and identify learning problems and offers some assignments that address these problems. This article briefly mentions some techniques that I have found helpful for working with the results and comments on the benefits of working with these assignments.

How Writing Assignments Expose Learning Problems

I use writing assignments continually in my classes with students of all levels. In one class of trigonometry students, a typical assignment had the immediate purpose of addressing the difficulties many students have in transferring from the visual to the verbal mode. The question was, "What is a reference angle?" An illustration and adequate definition of the concept are shown in figure 1.

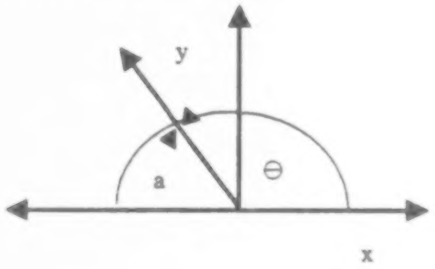

Fig. 1. An Illustration and adequate definition of a reference angle.

DEFINITION: A reference angle for a given angle ⊖ is an acute angle x that is formed by the X–axis and the terminal side of ⊖ (x is a reference angle for the angle ⊖.)

Whereas all the students could compute reference angles, only a few could adequately define the term. The students' difficulties fell into the following patterns:

1. Some students merely gave an example (which of course was a start:)

    - An angle like if you have 400 degrees on the unit circle your reference angle would be 40 degrees.

2. Others struggled for precision or failed to recognize the conditions of necessity and sufficiency that the definition requires:

    - Another angle that is the same as your first angle but in simpler terms

    (The words *same*, *first*, and *simpler* are vague, and what the "simpler terms" are is the essence of this definition.)

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- The acute angle with the same terminal side
- An angle closer to the x-axis than the original angle

3. Some tried to define simply by stating how the construction is used:
    - The angle that has to measure up supplementary with the angle given that tells which quadrant it is in
    - The angle that makes it easier to compute the trig functions for the first angle
4. Frequently students either were vague or said as much as possible, with the objective of maximizing partial credit on homework or a test:
    - An angle for which measurements of less than 90 are made in relationship to the x-axis only. The angle is smaller than the original and has equivalent trig functions, like 30, 60, etc., etc.
5. Surprisingly, very few students drew pictures, and when they did, the drawings were neither labeled nor referenced. Although drawing pictures may not be considered a part of writing definitions, students find it much easier to discuss or write about a concept if they have learned to draw an accurate picture first. Most students first grasp a concept visually or tactilely rather than verbally.

Another type of difficulty students have lies in using conventions of notation. Asking students to define a polynomial over the real numbers produces surprising results, even in higher-level calculus classes:

- A polynomial is $ax^2 + bx + c$.

  (This is only a second-degree polynomial. This student may never before have realized the need for the notation, $a_1, a_2, \ldots$.)

- $f(x)$ is a polynomial over the reals if $f(x) = a^{1}_{x} + a^{2}_{x} + \ldots$.

  (This student has remembered only that some notation is "up" and some "down")

Often the students presume that the reader already knows the answer and so an explanation or analysis is unnecessary. The assignment to describe the procedure for computing the determinant of a $x \times 3$ matrix generated the following responses:

- Take the row-1, column-1 position and multiply it by the determinant of the 2 x 2 matrix....
- Do not use the whole expansion, use co-factors, which is just telling what sign we give it. A plus or a minus times the minor.

The situation becomes more critical still when the question is not merely to state a definition or an algorithm but to write a proof. Then the problem becomes the students' lack of understanding of what constitutes a proof. The examples that follow are from an upper-level abstract algebra course:

- Question: Prove that if H and K are normal subgroups of a group G, H K is a normal subgroup of G.

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Answer: Since H and K are subgroups, H K is normal by a theorem.

- Question: Prove that 1cm (m, n) = mn/gcd(m, n.)

Answer: We must have that 1cm(m, n) - mn/gcd(m, n) because assume that this is not true. But that can't be because we know that 1cm(m, n) = mn/gcd(m, n.) Therefore this is a true statement.

Much energy in college teaching is dedicated to teaching advanced students to do proofs – questioning how to do them and wondering whether a proof is actually possible in certain situations. The algebra students whose work is shown in the foregoing may not belong in an upper-level course; they have reached this point only by mastering some well-digested problems. Their lack of understanding underscores the need for writing assignments at the earliest levels. Furthermore, the failure of many of our students successfully to create proofs or to write expositions raises questions about what they successfully construe or read and about the development of their learning skills. Frequently, students completely avoid reading the text and get by with methodically working exercises.

Activities That Address the Problems

Most of my explorative assignments are in-class summaries or assignments to be completed overnight; they are given to help the students understand a mathematical concept. They include the following types of exercises:

1. *Summaries.* Summaries can reveal where a class stands or serve as an introduction to a concept.
    - Right now, what is your notion of a "function?" Be as precise as possible: we say "$y = f(x)$ is a function of x if...."
    - What's wrong with this logic?

      (Example follows in a reducing-fractions problem)
    - What is a "radian," and why do we prefer this notion to that of "degree?"
2. *Visual image translation.* These experiences integrate visual and verbal understanding.
    - Describe the graph of this function as if you were explaining it to a friend over the telephone.
    - How are the graphs of $y = 1/x^2$ and $y = x^2$ related? How could you predict the behavior of the second from that of the first?
    - The class is divided into pairs. One student describes a graph in language that is as accurate and precise as possible while the other draws the graph. No peeking allowed on either side.
3. *Synopsizing tactics for solving a problem.* These exercises enhance students' awareness of problem-solving techniques.
    - Describe how to combine fractions to obtain one term. Do not merely give an example.
    - What is the difference between combinations and permutations and by what clues do you distinguish problems involving the one or the other?
4. *Giving an algorithm.* Students practice describing steps in algorithmic thinking.

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- Write out an explicit procedure that helps you to solve distance–time–rate problems.

5. *Giving a definition or stating a theorem.* Such verbalizations enhance the precision of students' terms and theorems.

6. *Communicating thoughts to a specific audience.* These exercises give students the experience of translating one explanation into another.

   - Many of you did not understand the concept on page 12 of the text. Rewrite that page as you would have liked to have seen it written.

   - In a few pages, write a "crib sheet" for your best friend, who has fallen seriously behind in this course. List important concepts, definitions, and theorems and briefly explain any methods of this unit on which the class might be tested.

7. Inventing a problem. Students learning to create their own illustrations through such exercises.

   - You live in a family with 3 sisters, 4 brothers, 2 dogs, and 17 cats. Create several problems that deal with combinations and permutations involving those numbers. Give answers and explain what allowed you to recognize that the problem was to be solved by the techniques you used.

8. *Generalizing a concept.* These exercises help to strengthen students' grasp of the overview of mathematics.

   - Mathematicians write definitions when a clear need arises. In your own study of polynomials, what definitions might you write to simplify talking about polynomials? Where could your definitions be useful?

   - Where in "real life" have you come across "absolute value?"

9. *Group projects.* Such projects enhance confidence and productivity by having students work collaboratively. These activities allow students to exchange ideas and to pursue the development of an idea in greater depth than they can when working alone. Since students enjoy this activity, it is a good way to introduce writing assignments.

   - Pass out a few transparencies and markers and have groups compete in writing an accurate statement of a theorem or an explanation of a procedure. Show the results on the overhead projector.

   - Have the groups anticipate a theorem. For instance, "A squirrel runs into a box and out again. It may enter through two doors and exit through three, so it can pass through the box in as many as six different ways. A very important principle is at work here – one that doesn't involve squirrels. Can you state it in its greatest generality?" Or, "Now that you have seen my explanations for finding maxima and minima visually, can you explain how the first derivative can indicate where a relative maximum or minimum occurs? If you can, you will have stated the *first derivative test.*"

Techniques and Benefits of Using Writing Assignments

Giving writing assignments, of course, does not in itself automatically generate effective teaching and learning; the teacher must respond to the writing. It is useful to approach writing as part of a process leading to completion in the future rather than as a final product; this approach shifts the perspective from what is wrong to how it could be made more accurate. Attempting to achieve this shift has led me to use several techniques.

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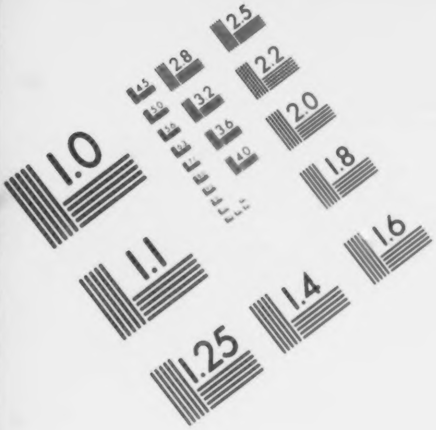
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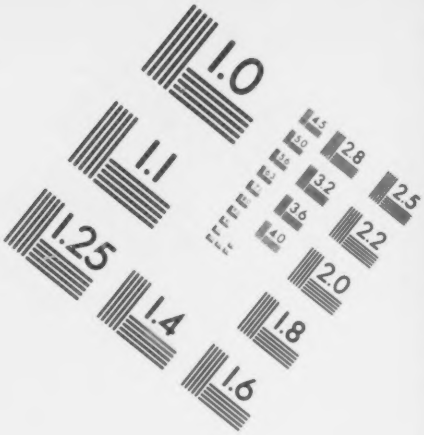
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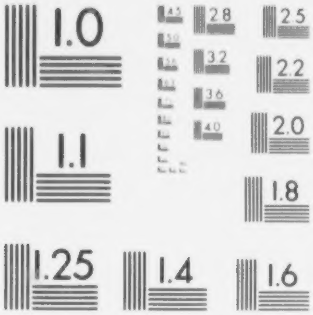
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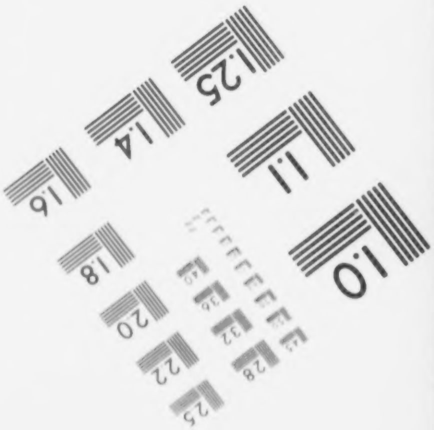

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1. *Discussion*. I put the students' efforts on transparencies, and show them on the overhead projector, encouraging discussion or giving my own reactions. In conformity with the NCTM's *Curriculum and evaluation Standards for School Mathematics*, which state that mathematics should be considered as a form of communication, every effort is made to understand each student's attempt. Any inclinations on the part of students to ridicule or be embarrassed generally subside when it becomes clear that I am trying to understand their thought processes and how their definitions might be improved. Something interesting and worthwhile is usually evident in every student's attempt, and, interestingly, students' answers often go awry in similar ways. The reactions of other students are useful in explaining what was intended; most of us have found that students sometimes have a mysterious ability to interpret the teacher's intent to other students.

2. *Peer evaluation*. Papers can be exchanged and evaluated according to specific criteria (see fig. 2 for a sample worksheet.) Such peer evaluation gives students an opportunity to see papers from a perspective closer to that of the teacher.

3. *Revision*. After the need for precision has been shown through peer evaluation, students may be asked to rewrite their definitions. Frequently, the entire exercise is repeated on another day. Students may think they understand a concept when the teacher shows the corrections on their work, but teachers can create no better learning opportunity for students than having them reconstruct their own answers in the correct way.

4. *Anticipation*. Quite often, the writing assignments generate lively discussion and can be used to introduce a topic; for instance, the assignment to define a degree can be followed by a discussion of the usefulness of radians.

These exercises are short and well worth the time they take. They teach strategies for maximizing understanding; they develop trust and improve participation and collaboration in and out of the classroom. Most students appreciate the chance to be heard and understood. From their point of view, the elegance of the text or the lectures may merely be obfuscating. Students are generally receptive to the discussion of class writing projects and to doing extra work at home because they can see the payoff in their enhanced understanding of the material. Students report that they feel they read and retain material more easily as mathematical conventions become familiar to them.

In the test setting, accuracy and formal quality are of course expected, but the students develop this quality by themselves as a result of their in–class experiences. Test results are frequently improved because detailed notice of a student's work sharpens the student's attention to detail.

I have found that I do not need to grade grammar and mechanics. My objective is to get closer to how students think, and continually correcting spelling and grammar tends to drive students into using a smaller vocabulary and taking fewer risks. In the students' writing generally, their control of grammar and mechanics is closely linked with their ability to use logic. The issue is simply whether they are adequately communicating with me, and the explicit grading, when done at all, is based on this criterion and goes very quickly.

Above all, the teacher can emphasize the fact that the writing of definitions, theorems, and later, proofs is a delightful experience. The aspect of elegance in our mathematical language can and should be shared by our students at every level.

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DEFINITION AND THEOREM WORKSHEET

DEFINITION WORKSHEET: NAME:______________________

DEFINE:

5 – The student has given a complete definition, with clear and accurate vocabulary and notation, appropriate mathematical vocabulary and minimum redundancy.

4 – Minor problems with clarity, notation, vocabulary or wordiness are evident.

3 – The basic concept is clear (student seems to know what is being defined,) but student exhibits significant confusion in notation or vocabulary.

2 – The basic concept shows significant confusion, with major problems in notation or vocabulary.

1 – The definition does not relate to the concept in question.

THEOREM WORKSHEET: NAME:______________________

STATE:

5 – The student has given a complete theorem, with clear and accurate vocabulary and notation, a complete hypothesis and conclusion, appropriate mathematical vocabulary and minimum redundancy.

4 – Minor problems arise with clarity, notation or vocabulary; or minor incompleteness is found in the hypothesis or conclusion.

3 – The basic concept of the theorem is clear (student seems to understand the theorem), but significant confusion or error is seen in vocabulary or notation.

2 – The basic concept of the theorem shows significant confusion about the hypothesis or conclusion, with major errors in notation or vocabulary.

1 – The theorem does not relate to the concept in question.

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APPENDIX E

GUIDELINES FOR EFFECTIVE MEDIA INTEGRATION

There is a growing collection of media resources that support the expectations of the Senior High Mathematics program. As with any resource, films and videos should be selected with due consideration for the expectations of the program and the needs of your class. Specific films and videos are noted throughout the Teacher Resource Manual.

The following questions should be considered when selecting media resources for use in the classroom.

- Does it support the expectations of the program?
- Does it meet individual student needs and learning styles?
- Does it reinforce, enrich or extend the program?
- How appropriate is the timing?
- Does the medium demonstrate abstract ideas?
- How well does the medium fit into existing curricula and teaching/learning needs?
- How clear are the educational aims and objectives?
- What kind of preparation is required?
- How are teaching and learning evaluated?
- What forms of follow-up are required?

Resource/Media Centres

There are 12 resource/media centres in Alberta that carry films and videos that support the Senior High Mathematics program. Each centre publishes its own catalogue, listing the resources in its collection. These centres operate as libraries, lending out audiovisual materials for specified time periods. For more information contact the local resource/media centre:

(A) Regional Resource Centres

Zone 1
Zone One Regional Film Centre
P.O. Box 6536
10020 - 101 Street
Peace River, Alberta
T8S 1S3
Telephone: (403) 624–3187

Zone 2/3
Central Alberta Media Services
(CAMS)
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: (403) 464–5540 or 467–8896

Zone 4
Alberta Central Regional Educational Services
(ACRES)
County of Lacombe
Box 3220
5140 - 49 Street
Lacombe, Alberta
T0C 1S0
Telephone: (403) 782–5730

Zone 5
South Central Alberta Film Federation
(SCAFF)
Westmount School
Box 90
Wheatland Trail
Strathmore, Alberta
T0J 3H0
Telephone: (403) 934–5028

Zone 6
Southern Alberta Regional Film Centre
(SAFRC)
McNally School
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: (403) 320–7807

(B) Urban Media Centres

County of Strathcona
Learning Resource Service
2001 Sherwood Drive
Sherwood Park, Alberta
T8A 3W7
Telephone: (403) 464–8235

Calgary Separate School Board
Instructional Materials
6220 Lakeview Drive S.W.
Calgary, Alberta
T3E 6T1
Telephone: (403) 246–6663

Edmonton Public School Board
Learning Resources Centre
Centre for Education
One Kingsway
Edmonton, Alberta
T5H 3G9
Telephone: (403) 429–8320

Edmonton Catholic Schools
Curricular Resources
St. Anthony's Teacher Centre
10425 - 84 Avenue
Edmonton, Alberta
T6E 2H3
Telephone: (403) 439–7356

Red Deer Public School Board
4747 - 53 Street
Red Deer, Alberta
T4N 2E6

Telephone: (403) 343–1405

Calgary Board of Education
Education Media
3610 - 9th Street S.E.
Calgary, Alberta
T2G 3C5
Telephone: (403) 294–8540

Medicine Hat School District
Instructional Materials Centre
601 First Avenue S.W.
Medicine Hat, Alberta
T1A 4Y7

Telephone: (403) 526–1323

APPENDIX F

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9945 - 50 Street
Edmonton, Alberta T6A 0L4
427-5782

Fort McMurray

Fort McMurray Provincial Building
5th Floor, West Tower
9915 Franklin Avenue
Fort McMurray, Alberta T9H 2K4
743-7231

Grande Prairie

Grande Prairie Provincial Building
10320 - 99 Street, Box 7
Grande Prairie, Alberta, T8V 6J4

Lethbridge

300 Professional Building Bag Service 3014
740 - 4 Avenue South
Lethbridge, Alberta, T1J 4C7
381-5360

Medicine Hat

303 Provincial Building
770 6 Street S.W.
Medicine Hat, Alberta, T1J 4J6
529-3535

Peace River

Provincial Building
9621 - 96 Avenue
Bag 900, Box 9
Peace River, Alberta, T0H 2X0
624-6214

Red Deer

2nd Floor, Provincial Building
4920 - 51 Street
Red Deer, Alberta, T4N 6K8
340-5241

Alberta Consumer and Corporate Affairs

Box 1616
Edmonton, Alberta, T5J 2N9

APPENDIX G

ALBERTA EDUCATION MATHEMATICS CONSULTANTS

Grande Prairie Regional Office

Hugh Sanders
5th Floor, Nordic Court
10014 - 99 Street
Grande Prairie, Alberta T8V 3N4
Phone: 538-5130
FAX: 538-5135

Edmonton Regional Office

Dick Daly
Devonian Building - West Tower
11160 Jasper Avenue
Edmonton, Alberta T5K 0L2
Phone: 427-2952
FAX: 422-9682

Art Peddicord
Devonian Building - West Tower
11160 Jasper Avenue
Edmonton, Alberta T5K 0L2
Phone: 427-2952
FAX: 422-9682

Red Deer Regional Office

Ron Babiuk
3rd Floor West, Provincial Building
4920 - 51 Street
Red Deer, Alberta T4N 6K8
Phone: 340-5262
FAX: 340-5305

Calgary Regional Office

Pat McLaughlin
Room 1200, Rocky Mountain Plaza
615 Macleod Trail, SE
Calgary, Alberta T2G 4T8
Phone: 297-6353
FAX: 297-3842

Lethbridge Regional Office

Gary Hill
Provincial Building
200 - 5th Avenue, South
Lethbridge, Alberta T1J 4C7
Phone: 381-5243
FAX: 381-5734

END

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